ISSN 1992-6502 (Print)
Vol. 17, no. 6 (59), pp. 70-74, 2013.
Vestnik UGATU
ISSN 2225-2789 (Online)
http://journal.ugatu.ac.ru

UDC 004.7

THREE-DIMENSIONAL LOADING VEHICLE ROUTING PROBLEM SOLUTION WITH SET-PARTITIONING-BASED METHOD

N. GILMANOVA[1], E. BRONSHTEIN[2]

[1] hopediesthelast@yahoo.com, [2] bro-efim@yandex.ru

Ufa State Aviation Technical University, Russian Federation

Submitted 2013, june 16

Abstract. The article considers the optimization problem of vehicle routing with three-dimensional loading constraints. Several practical loading constraints encountered in freight transportation are formalized. The efficiency of using the set-partitioning approach to improve heuristic solution is shown by means of computational experiment.

Keywords: vehicle routing, three-dimensional loading, heuristics, set-partitioning.

1. INTRODUCTION

Freight transportation planning is an integral part of the logistic complex of manufacturing and trade enterprises. The article considers planning of the delivery of goods in containers (boxes) from the warehouse of the small-scale wholesale trade enterprise to the number of clients. This delivery process is the final link in the supply chain. Our objective is to find a transportation plan of the lowest cost which takes into account various technological constraints on goods loading in vehicles.

Having a limited number of vehicles, the complexity of the delivery planning stage is explained by the fact that there may be no obvious feasible transportation options meeting all clients' demands. Not only the search for the optimal delivery plan, but even finding any feasible ones to have a base for decision making may become a difficult problem.

In practice, the delivery planning is often carried out manually by experts who assign vehicles for delivering goods to the stable "zones" of customers' location. The main indicators here are cost of product per 1 km, minimal cost of product shipped by a single vehicle and others. However such indicators do not ensure feasibility of the delivery plan and do not evaluate efficiency of transportation for the whole enterprise rather then for a single vehicle. That is why delivery planning requires development of specialized systems that make use of optimization tools.

2. FREIGHT DELIVERY PLANNING AS OPTIMIZATION PROBLEM

There are three main modules of production delivery planning: vehicle selection, routing, and loading scheme construction. Having a common goal – overall transportation costs minimization – these modules require taking into account various technological constraints, such as the listed below.

Vehicle selection:

- Lifting capacity;
- Volume of the cargo hold;
- Number of the vehicles available;
- Fixed hiring cost;
- Cost per 1 km or 1 hour etc.

Routing:

- Maximum length of the route (duration of transportation);
- Maximum number of customers per route etc.

Vehicle loading scheme construction:

- Orientation of the loaded items;
- Fragility of the items;
- Stability of placement;
- Loading/unloading sequence etc.

During the delivery planning these modules should be considered as a whole because of their multilateral relationship. For instance, it may be possible to load some subset of items corresponding to some route in one vehicle and impossible to load it in another vehicle with smaller capacity (so the feasibility of a route depends on the vehicle selection). In turn, loading scheme should take into account the specified route since when visiting certain customer, the items ordered by this customer should be available for unloading and not be blocked by the items ordered by the following customers. If some vehicle is selected then there may exist a feasible loading scheme for one sequence of customers visiting and no feasible schemes for another sequence, even more preferable by the travelling distance criteria (so the feasibility of loading scheme depends on the route).

The wide class of vehicle routing problems (VRP) includes a variety of models that take into account various conditions motivated by the real practice. All these problems have the same goal: to construct the minimum cost set of routes for the vehicles delivering goods ordered by a number of customers.

In transportation process much attention should be paid to the efficient utilization of vehicle space. If the cargo hold has a form of parallelepiped and the goods are stored in relatively small containers or boxes then the problem of effective utilization of vehicle space may be reduced to constructing a scheme of orthogonal placement of the items inside the vehicle, i.e. to the three-dimensional bin packing or container loading problem.

The most full picture of the situation is given by the vehicle routing problem with three-dimensional loading constraints (3L-CVRP). It combines the capacitated vehicle routing problem (CVRP) and the three-dimensional bin packing problem (3DBPP). 3L-CVRP was first formulated by Gendreau et al. [1] and requires defining a minimum cost set of routes for the fleet of identical vehicles to deliver production in boxes or containers to a set of customers. Each route should be provided with a feasible scheme of items loading in the vehicle. An example of 3L-CVRP solution is shown in Fig. 1.

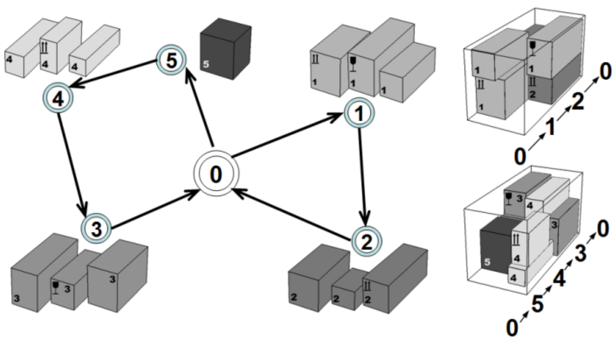

Fig. 1. Example of the 3L-CVRP

The computational complexity of the 3L-CVRP is extremely high, significantly exceeding the complexity of NP-hard 3DBPP and CVRP. Thus, 3L-CVRP is one of the most challenging combinatorial optimization problems.

3. PROBLEM DESCRIPTION

The problem is considered in the following formulation.

Let $G=(V,E)$ be an undirected graph with set of vertices $V=\{0,1,\ldots,n\}$, where vertex 0 represents the depot (warehouse) and vertices 1,…,*n* represent the customers; and set of edges $E=\{[i,j]:i,j\in V\}$; c_{ij} denotes the distance between vertices *i* and *j*. At the disposal of the company there are *T* identical vehicles having lifting capacity *M*, length of the cargo hold *L*, width *W*, height *H*, and the fixed hiring cost *C*. Each customer *i* (*i*=1,…,*n*) orders a set of q^i items (rectangular boxes). Each item *k* $(k=1,\ldots,q^i)$ has its length l_k^i, width w_k^i, height h_k^i, weight m_k^i, fragility mark $f_k^i\in\{0,1\}$ (if $f_k^i=1$ then the item is fragile), and the set of allowable orientations $r_k^i=[r_{k,1}^i,\ldots,r_{k,6}^i]$, where $r_{k,j}^i\in\{0,1\}$ denotes the allowance of placing item *k* of customer *i* in orientation *j*=1,…,6 (there are six possible orientations ensuring the orthogonality of loading, with the sides of the items parallel to the sides of the vehicle).

The objective is to find a set of delivery routes with the minimum overall cost comprising the fixed hiring costs and costs depending on the distance traveled. Each single-vehicle route should be started and finished at the depot; the number of vehicles used should not exceed *T*. All the items ordered by the single customer should be delivered by the single vehicle (order splitting is not allowed). Total weight of the goods carried by the single vehicle should not exceed the lifting capacity. For each vehicle the items loading scheme should be determined.

Let us consider the route $R=(0,p_1,\ldots,p_i,\ldots,p_Q,0)$ $(p_i\in\{1,\ldots,n\})$ passing through *Q* customers. The following constraints determine the feasibility of the loading scheme for the vehicle along this route:

- each item is in allowable orientation;
- items do not overlap with each other and with the sides of the vehicle;
- the bottom side of the non-fragile item has no direct contact with the upper sides of the fragile items;
- the loading of the items is stable;
- at each point of the route the items ordered by corresponding customer are accessible for un-

loading and there is no need to move the items ordered by other customers (LIFO policy).

The scheme of items loading in the vehicle is denoted by $(x_k^{p_i}, y_k^{p_i}, z_k^{p_i}, \chi_k^{p_i}, \psi_k^{p_i}, \zeta_k^{p_i})$, $i=1,\dots,Q$, $k=1,\dots,q^{p_i}$, where q^{p_i} is the number of items ordered by the customer p_i, $x_k^{p_i}, y_k^{p_i}, z_k^{p_i}$ are the coordinates of further bottom left corner of the item *k*, and $\chi_k^{p_i}, \psi_k^{p_i}, \zeta_k^{p_i}$ are the lengths of projections of this item on the coordinate axis defining the orientation of the item. The route is feasible if it satisfies all the following constraints:

$$\sum_{i=1}^{Q}\sum_{k=1}^{q^{p_i}} m_k^{p_i} \le M, \qquad (1)$$

$$\left[\begin{array}{l} \left(r_{k,1}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=l_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=w_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=h_k^{p_i}\right),\\ \left(r_{k,2}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=w_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=l_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=h_k^{p_i}\right),\\ \left(r_{k,3}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=h_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=l_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=w_k^{p_i}\right),\\ \left(r_{k,4}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=l_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=h_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=w_k^{p_i}\right),\\ \left(r_{k,5}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=w_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=h_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=l_k^{p_i}\right),\\ \left(r_{k,6}^{p_i}=1\right)\wedge\left(\chi_k^{p_i}=h_k^{p_i}\right)\wedge\left(\psi_k^{p_i}=w_k^{p_i}\right)\wedge\left(\zeta_k^{p_i}=l_k^{p_i}\right); \end{array}\right. \qquad (2)$$

$i=1,\dots,Q$, $k=1,\dots,q^{p_i}$;

$$\left\{\begin{array}{l} \left(x_k^{p_i}\ge 0\right)\wedge\left(y_k^{p_i}\ge 0\right)\wedge\left(z_k^{p_i}\ge 0\right),\\ \left(x_k^{p_i}+\chi_k^{p_i}\le L\right)\wedge\left(y_k^{p_i}+\psi_k^{p_i}\le W\right)\wedge\left(z_k^{p_i}+\zeta_k^{p_i}\le H\right); \end{array}\right. \qquad (3)$$

$i=1,\dots,Q$, $k=1,\dots,q^{p_i}$;

$$\left[\begin{array}{l} (x_k^{p_i}\ge x_t^{p_j}+\chi_t^{p_j})\wedge(i\le j),\\ (x_t^{p_j}\ge x_k^{p_i}+\chi_k^{p_i})\wedge(j\le i),\\ y_k^{p_i}\ge y_t^{p_j}+\psi_t^{p_j},\\ y_t^{p_j}\ge y_k^{p_i}+\psi_k^{p_i},\\ z_k^{p_i}> z_t^{p_j}+\zeta_t^{p_j},\\ z_t^{p_j}> z_k^{p_i}+\zeta_k^{p_i},\\ (z_k^{p_i}=z_t^{p_j}+\zeta_t^{p_j})\wedge(i\le j)\wedge((f_k^{p_i}=1)\vee(f_t^{p_j}=0)),\\ (z_t^{p_j}=z_k^{p_i}+\zeta_k^{p_i})\wedge(j\le i)\wedge((f_k^{p_i}=0)\vee(f_t^{p_j}=1)); \end{array}\right. \qquad (4)$$

$i, j=1,\dots,Q; k=1,\dots,q^{p_i}, t=1,\dots,q^{p_j}, k\ne t$ при $i=j$;

$$\left[\begin{array}{l} z_k^{p_i}=0,\\ (x_k^{p_i}+\dfrac{\chi_k^{p_i}}{2}; y_k^{p_i}+\dfrac{\psi_k^{p_i}}{2})\in conv\ S; \end{array}\right. \qquad (5)$$

$i=1,\dots,Q$, $k=1,\dots,q^{p_i}$;

$S=\{(x_S, y_S)\}:\ \forall z\in(0, z_k^{p_i})\ \exists\ j,t\ (t\in 1,\dots,q^{p_j}):$

$$\left\{\begin{array}{l} (z_t^{p_j}\le z)\wedge(z_t^{p_j}+\zeta_t^{p_j}>z),\\ (x_t^{p_j}\le x_S)\wedge(x_t^{p_j}+\chi_t^{p_j}>x_S),\\ (y_t^{p_j}\le y_S)\wedge(y_t^{p_j}+\psi_t^{p_j}>y_S). \end{array}\right.$$

Constraint (1) ensures that the total weight of items loaded does not exceed the lifting capacity of the vehicle; (2) guarantees the feasibility of each items' orientation; (3) prohibits to place items outside the vehicle. Constraint (4) ensures compliance of three restrictions for each pair of items: overlapping prohibition, LIFO policy, and the fragility restriction. Finally, (5) is responsible for the stability of the items loading: if the item is placed not over the bottom of the vehicle but on the tops of the other items than the projection of the center of gravity of the item on a plane parallel to the vehicle bottom must be in a convex hull of area *S* which is defined by the union of the intersections of projections of the lower "supporting" items on this plane. Fig.2 depicts an example of checking whether placing item *k* on the items t_1 and t_2 is stable. The projection of the center of gravity of the item *k* belongs to the convex hull of supporting area *S* (encircled by a dotted line), thus placing is stable.

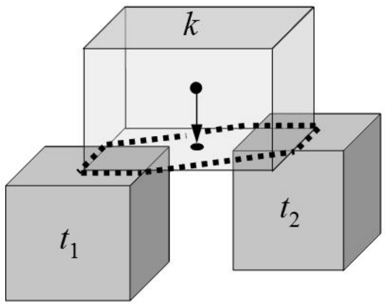

Fig. 2. Checking the stability of items placement

Assume there is a set of feasible routes $\Re$, c_R is the cost of route $R\in\Re$ defined by its length . The parameter $a_{iR}\in\{0,1\}$ equals 1 if route R includes customer *i* (*i*=1,…,*n*) and 0 otherwise. Variable ξ_R equals 1 if route R is included into a set of routes representing the current solution of the problem. Then 3L-CVRP is formulated as follows:

$$\sum_{R\in\Re} a_{iR}\xi_R = 1 \quad (i=1,\dots,n), \qquad (6)$$

$$\sum_{R\in\Re}\xi_R \le T, \qquad (7)$$

$$\xi_R\in\{0,1\} \quad (R\in\Re), \qquad (8)$$

$$\sum_{R\in\Re}(c_R+C)\xi_R \to \min. \qquad (9)$$

According to (6), each customer is visited along a single route, (7) establishes the number of routes not exceeding the number of available vehicles *T*, (8) ensures variables ξ_R to be binary, objective function (9) minimizes overall cost of transportation.

4. APPROACHES FOR THE 3L-CVRP SOLUTION

To solve the 3L-CVRP, a number of approaches was proposed including tabu search in several variants [1–4] and ant colony optimization [5]. We should note that the authors listed above solved 3L-CVRP in a formulation slightly different from that presented in this article:

- a fixed set of orientations was considered to be feasible for all items (boxes can be rotated by 90° on the width-length plane but cannot be turned over);
- the stability of placement was evaluated supposing the supporting area (area where the bottom of upper item touches the tops of supporting items) to be not less then 75 % of the upper items' bottom area;
- objective function did not include the fixed hiring costs that depend only on the number of vehicles used and do not depend on the travelled distance.

For solving the 3L-CVRP we proposed a heuristic method of three-phase decomposition [6]. The method was named after "two-phase decomposition", a heuristic method of solving classical CVRP that has two main procedures: partitioning of the customers set into single-vehicle-served zones (clusters) and routing on each zone [7]. The third procedure of the three-phase decomposition is search for a feasible vehicle loading scheme according to given route. For that purpose we use a multi-method algorithm 3D-MMA [8] embedded into one-point evolutionary algorithm (1+1)-MMEA. Thus, there are three main procedures: zoning, routing, and loading.

In this paper we propose to supplement the 3L-CVRP solving by the procedure of solution improvement. Procedure consists in defining an optimal (regarding to (9)) subset of routes given a set of feasible routes found during the problem solution by any method. The issue is that any heuristic method somehow looks over various routes, defining if they are feasible, and selects some subset of feasible routes to be the final problem solution. However, in the set of feasible routes there can be a subset of the least cost, where the routes have been found at different moments of solution process and have not been included into the final heuristic solution. To define this subset we have to solve a problem of optimal set-partitioning, diving a set of clients into the single-vehicle routes on the base of feasible routes set.

It should be noticed that generally the set-partitioning formulation of the routing problems is considered to be impractical. Nevertheless, due to the large number of constraints, this approach is useful for the 3L-CVRP because the number of feasible routes is not as large as in most other routing problems.

Moreover, the problems of low dimensionality (defined primarily by the number of clients and the number of items in each order) may be solved using a set-partitioning approach on the extended set of feasible routes constructed by the exhaustive search. It is necessary to enumerate all possible subsets of customers having total items weight not exceeding the lifting capacity of the vehicle and total items volume not exceeding the cargo hold volume. For each of these subsets a travelling salesman problem should be solved to find the shortest route with the feasible loading scheme.

For the problems of higher dimensionality constructing an extended set of feasible routes requires huge computational resources so the feasible routes set may be obtained during heuristic solution (for example, using three-phase decomposition method). To solve the 3L-CVRP in formulation (6)–(9) having a feasible routes set we implement a simple algorithm by [9] that uses a search tree.

5. COMPUTATIONAL EXPERIMENT

The efficiency of using an optimal customer-set-partitioning algorithm having a set of feasible routes obtained during solution of 3L-CVRP by three-phase decomposition method is confirmed by the computational experiment. We used the instances proposed by Gendreau et al. [1]. Each instance in a formulation described in chapter 3 was being solved by three-phase decomposition method within one minute with the following parameters:

- not more than 10 shortest routes on one zone checked for loading feasibility;
- not more than 1000 iterations of (1+1)-MMEA during the feasible loading search.

The solution obtained was fixed. Then on the base of the feasible routes set obtained we implemented an optimal set-partitioning algorithm and evaluated the gap from heuristic solution. The results are shown in Table 1.

Improvement of heuristic solution was obtained on 12 instances of 25 after using of set-partitioning procedure. In some cases the improvement is quite significant (up to 9 %). This result assures the efficiency of implementing a set-partitioning procedure given a set of feasible routes obtained heuristically.

Table 1. Results of computational experiment

No.	Number of clients; items; vehicles available	Overall routes cost		Gap, %
		Three-phase decomposition only	After set-partitioning	
1	15; 32; 5	302,0	301,7	-0,1
2	15; 26; 5	335,0	335,0	0
3	20; 37; 5	387,4	387,4	0
4	20; 36; 6	440,7	440,7	0
5	21; 45; 7	535,5	535,5	0
6	21; 40; 6	498,3	498,3	0
7	22; 46; 6	788,4	788,4	0
8	22; 43; 8	826,4	812,8	-1,65
9	25; 50; 8	665,4	665,4	0
10	29; 62; 10	955,4	868,6	-9,09
11	29; 58; 9	841,3	828,3	-1,55
12	30; 63; 9	625,1	619,9	-0,83
13	32; 61; 9	2895,7	2878,0	-0,61
14	32; 72; 11	1545,0	1502,0	-2,78
15	32; 68; 10	1468,0	1416,6	-3,5
16	35; 63; 11	706,2	698,6	-1,07
17	40; 79; 14	916,4	904,8	-1,27
18	44; 94; 14	1243,6	1243,6	0
19	50; 99; 13	914,2	862,9	-5,61
20	71; 147; 20	769,3	769,3	0
21	75; 155; 18	1300,9	1300,9	0
22	75; 146; 19	1439,7	1439,7	0
23	75; 150; 18	1290,1	1290,1	0
24	75; 143; 18	1336,8	1266,5	-5,26
25	100; 193; 24	1684,7	1684,7	0
Average		**988,5**	**974,4**	**-1,33**

6. CONCLUSION

In this paper the following main results are introduced:

- a detailed description of the 3L-CVRP with the formalization of technical constraints;
- a new formulation of the stability constraint based on the center of gravity of each item;
- a procedure of heuristic solution improvement using the set partitioning algorithm with the experimental rationale.

REFERENCES

1. M. Gendreau, M. Iori, G. Laporte, and S. Martello, "A tabu search algorithm for a routing and container loading problem," *Transportation Science*, 40 (3), pp. 342–350, 2006.
2. C. Tarantilis, E. Zachariadis, and C. Kiranoudis, "A hybrid metaheuristic algorithm for the integrated vehicle routing and three-dimensional container-loading problem," *Intelligent Transportation Systems,* 10 (2), pp. 255–271, 2009.
3. A. Bortfeld, "A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints", *Computers & OR,* 39(9), pp. 2248-2257, 2012.
4. L. Wang, S. Guo, S. Chen, W. Zhu, and A. Lim, "Two natural heuristics for 3D packing with practical loading constraints," *Lecture Notes in Computer Science,* 6230, pp. 256–267, 2010.
5. G. Fuellerer, K. Doerner, R. Hartl, and M. Iori, "Metaheuristics for vehicle routing problems with three-dimensional loading constraints", *European Journal of Operational Research,* 201, pp. 201–751, 2010.
6. N. A. Gilmanova, "A three-phase decomposition method for the capacitated vehicle routing problem with loading constraints," (in Russian), *St.Petersburg State Politechnical University Journal,* 5 (133), pp. 35–42, 2011.
7. M. L. Fisher and R. Jaikumar, "A generalized assignment heuristic for the vehicle routing problem," *Networks,* 11, pp. 109–124, 1981.
8. Yu. I. Valiahmetova, E. A. Mukhacheva, A. S. Filippova, N. A. Gilmanova, and U. A. Karipov, "Multi-method technology of orthogonal packing and its application to transport logistic problems," *Information technologies,* 12: Supplement, pp. 2-32, 2010.
9. R. S. Garfinkel and G. L. Nemhauser, "The set partitioning problem: set covering with equality constraints", *Operational Research,* 17, p. 848, 1969.

ABOUT AUTHORS

Gilmanova, Nadezda Aleksandrovna, Chief Expert, OJSC "URALSIB". Dipl. Economist-Mathematician (UGATU, 2009).

Bronshtein, Efim Mikhaylovich, Prof., Dept. of Computatuinal Mathematics and Cybernetics. Dipl. Engineer (Ufa Aviation Inst., 1968). Cand. (Phd) of Phys. & Math. Sci. (Inst. of Math., Novosibirsk, 1980), Dr. (Habil.) of Phys. & Math. Sci. (Bashkir State Univ., 1999).