

PRACTICAL APPLICATION OF POPULATION BASED ANT COLONY OPTIMIZATION ALGORITHM

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Abstract. In this paper we consider the Split Delivery Vehicle Routing Problem, which has a wide practical application. The SDVRP is NP-hard problem. We propose a population based ant colony optimization algorithm for solving the SDVRP. Computational experiments for developed algorithm are reported.

Keywords: Routing; Split Delivery Vehicle Routing Problem; Population Based Ant Colony Optimization Algorithm; pheromone.

1. INTRODUCTION

At the present time due to the increased of freight traffic the actual problem is the development of methods for solving the Vehicle Routing Problems, the main objective of which – reducing costs of transportation and delivery of different goods to customers «just in time». Vehicle Routing Problems are key in areas of transportation, relocation and logistics.

The problem of delivery of equipment for building and service of base stations (BSs) of JSC «VimpelCom» company in the republic of Bashkortostan is considered. This problem belongs to a subclass of Vehicle Routing Problems, called Split Delivery Vehicle Routing Problems, SDVRP [2].

SDVRP is considered in many sources, because it has a very wide practical application in various fields of human activity [2]. This problem is a combinatorial problem of discrete optimization which falls into the category of NP-hard problems, meaning that the computational effort required to solve this problem increases exponentially with the problem size. Exact solutions of such problems may be reached only by using search algorithms, which require unacceptable much time. For such problems it is often desirable to obtain approximate solutions, so they can found fat enough and are sufficiently accurate for the purpose. Therefore usually these problems are solving by using various heuristic methods, which rely on some insight into the problem nature.

Population Based Ant Colony Optimization (P-ACO) algorithm is widely discussed in literature [1, 3–6]. This algorithm was used to solving different problem [4–7].

In this paper we consider the SDVRP, for the solving of which is used P-ACO algorithm.

The paper is organized as follows. In section 2 we provide the problem formulation. In section 4 we present the proposed method for solution. It also includes the experiments in Section 5, the analysis and discussion of the results in Section 6.

2. A MATHEMATICAL FORMULATION

Mathematically [2] the SDVRP can be defined over a weighted graph $G=(V,A)$ with vertex set $V=\{v_0,v_1,\dots,v_n\}$ and edge set $A=\{(v_i,v_j)/v_i,v_j\in V;i\neq j\}$. Depot is represented by a vertex v_0 , vertex set $V'=V\setminus\{v_0\}$ represent n cities and Bss. The traversal cost (also called length) c_{ij} of an edge $(i,j)\in A$ is supposed to be non-negative. An integer non-negative demand d_i is associated with each BS $v_i\in V'$. R_v is the route of vehicle v , each vehicle has a capacity $Q\in Z^+$.

For the problem are given the following constraints: 1) the route of each vehicle has to start at finish at the depot v_0 ; 2) the demands of the BSs in equipment must be satisfied; 3) the total length of the route is minimal.

We use the following notations:

x_{ij}^v is a Boolean variable which is equal to 1 if vehicle v travels directly from i to j , and to 0 otherwise,

y_{iv} is the quantity of the demand of I delivered by the v -th vehicle.

The SDVRP can now be formulated as follows:

$$\min \sum_{i=0}^n \sum_{j=0}^n \sum_{v=1}^m c_{ij} x_{ij}^v \quad (1)$$

subject to:

$$\sum_{i=0}^n \sum_{v=1}^m x_{ij}^v \geq 1 \quad j = 0, \dots, n, \quad (2)$$

$$\sum_{i=0}^n x_{ip}^v - \sum_{j=0}^n x_{pj}^v = 0 \quad p = 0, \dots, n; v = 1, \dots, m, \quad (3)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^v \leq |S| - 1 \quad v = 1, \dots, m; S \subseteq V - \{0\}, \quad (4)$$

$$y_{iv} \leq d_i \sum_{j=0}^n x_{ij}^v \quad i = 1, \dots, n; v = 1, \dots, m, \quad (5)$$

$$\sum_{v=1}^m y_{iv} = d_i \quad i = 1, \dots, n, \quad (6)$$

$$\sum_{i=1}^n y_{iv} \leq Q \quad v = 1, \dots, m, \quad (7)$$

$$x_{ij}^v \in \{0, 1\} \quad i = 0, \dots, n; j = 0, \dots, n; v = 1, \dots, m, \quad (8)$$

$$y_{iv} \geq 0 \quad i = 1, \dots, n; v = 1, \dots, m. \quad (9)$$

Constraints (2)–(4) are the classical routing constraints: constraints (2) impose that each vertex is visited at least once, (3) are the flow conservation constraints while (4) are the subtours elimination constraints. Constraints (5)–(7) concern the allocation of the demands of the customers among the vehicles: constraints (5) impose that customer i can be served by vehicle v only if v passes through i , constraints (6) ensure that the entire demand of each vertex is satisfied, while constraints (7) impose that the quantity delivered by each vehicle does not exceed the capacity.

3. POPULATION BASED ANT COLONY OPTIMIZATION ALGORITHM

Population Based Ant Colony Optimization (P-ACO) algorithm is a part of a large class of algorithms of swam intelligence, in which for solving the problem are used the peculiarities of behavior of social insects such as bees and ants, – independent artificial agents, each of which construct a feasible solution of discrete optimization problem on the base of accumulation and using static information – artificial traces of pheromone and heuristic problem-specific information.

P-ACO algorithm is based on the ability of ants to find the shortest path to a food source and to adapt to changing conditions. Ants communicate with each other by products of external secretion – pheromone, a fixed amount of which they lay on their path. The ant, who has found a shorter route, will be enrich a pheromone trace more intensively. When choosing the travel direction the ants take into account the intensity of pheromone trace, so, a shorter path gets advantage. Evaporation of pheromone and an element of chance in choosing the path allow get out of local optimum and to find the shortest routes.

The basic idea of P-ACO algorithm consists in transmission to the following iteration only the most important and lesser on volume information. This idea is realized by creating a population P of the best solutions obtained by the agents on t iterations. Then, these solutions are used to calculate the level of pheromone [6].

4. APPLICATION OF P-ACO ALGORITHM FOR SOLVING THE SDVRP

At solving the SDVRP the agents imitate the vehicles, routes are constructed step by step by selection of the next customer until all customers will be served. Initially, the agent starts from the depot and a list of customers, included in its route, is empty. The agent selects the next customer from the list of available customers and updates its current load before the next selection. The agent returns to the depot, when its maximum capacity has been reached or all customers already have been visited. The total length L_l is calculated as the value of objective function of a complete route of l -th artificial agent. P-ACO algorithm constructs a complete route for the first ant before the second agent will begin its motion. This continues until a predetermined number of agents m haven't constructed a full route.

The general scheme of P-ACO-SDVRP algorithm is presented below:

Input parameters: a number of cities and BSs n , a number of vehicles m , a matrix of capacities of vehicles Q_m , a distance matrix $C = \{c_{ij}\}$, the algorithm parameters α, β : a pheromone matrix: a matrix of demands of BSs d_n ; a number of agents l .

Output parameters: a matrix of population P (l best found solutions).

$P := \emptyset$

Repeat

For each agent $l \in [1, \dots, t]$ {construction of solution}

$S := \{1, \dots, n\}$ {the set of given cities and BSs}

$Tour^l := \emptyset$

$init := 0$ (the depot)

$Tour^l := Tour^l + \{init\}$

For $i := 1$ to n **do**

Select the city of the BS $next$ with probability $p_{init,next}$ by the formula (1)

If $next$ is the BS **then begin**

If $(Q - d_{init} \leq 0), Q = Q_v$, **then begin**

Another vehicle $v \in [1, \dots, m]$ with capacity $Q := Q_v \geq q_{next}$ continues to serve the BS $next$ and the demand in equipment of $next$ -th BS is satisfied

End If

Else begin

The previous vehicle continues to serve the *next*-th BS

End Else

End If

If the demand of the BS *next* is satisfied **then begin**

The BS *next* is removed from the set *S*:

$S := S - \{next\}$

$Init := next$

End If

$Tour^l := Tour^l + \{init\}$

If ($S = \emptyset$) **then begin**

Break;

End If

Calculate the length L^l of the $Tour^l$

End For

Call the algorithm CALCULATION_THE_MATRIX_OF_PHEROMONE

Until end condition

Select the best solution from the matrix of population *P*

End P-ACO-SDVRP

CALCULATION_THE_MATRIX_OF_PHEROMONE

Input parameters: a number of agents *l*, a matrix of population *P* (*l* best found solutions).

Output parameters: an updated matrix of pheromone *P*.

If $|P| = t$ **then begin**

Remove the most «old» solution $\tilde{\pi}$ from the population:

$P := P - \tilde{\pi}$

Determine the best solution of iteration and add it to the population:

$P := P + \pi^*$

Calculate a new pheromone matrix from *P*.

End If

We use the following stopping criterion: if $N = const$, where *N* is the number of steps, we suppose, that the stopping criterion is fulfilled.

The probability of choosing a city *j* at city *i* is defined as

$$p_{ij}^k = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N^k} [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta}, & \text{ecnu } j \in N^k, \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

where N^k is the vertex set not yet visited by the agent *k*; $\eta_{ij} = \frac{1}{d_{ij}}$ is heuristic information; $\tau_{ij}(t)$,

where *t* is the number of iteration of P-ACO-SDVRP algorithm, is the pheromone level, which shows the «desirability» of transition into a city *j* from the city *i*, in other words, the desirability of the appearance of the edge (*i*, *j*) in the solution; pa-

rameters α and β define the relative influence between the heuristic information and the pheromone levels. At $\alpha = 0$ the algorithm degenerates into greedy, when the nearest city is chosen without accounting of quantity of pheromone.

5. COMPUTATIONAL EXPERIMENTS

Three benchmark instances were selected for computational experiments from international test library OR-library. As test system the 2. workstation with 2-nuclear processor Intel Core 2 Duo GHz and 2 GB of operative memory were used.

Influence of the values of the parameters α and β on the quality of the solution was identified during the numerical experiment.

The problem with 55 customers pr01 was chosen for the first three test collections. The best known solution for this problem is equal to 524.61.

Influence of the parameter α was verified in the first test collection (α is the parameter of consequence of pheromone amount during selection of the next customer). The value of the parameter α was varied from 1 to 10, the others parameters were fixed: $\beta = 1, p = 0.2, O = 524$. Table 1 shows that the increase of the parameter α leads to considerable worsening of the solutions.

Table 1

Influence of the parameter α on the quality of the solution *L*

α	1	3	5	7	9
<i>L</i>	583.4	771.2	1221.3	1109.4	1144.8

Influence of the parameter β was verified in the second test collection (β is the parameter of consequence of distance during selection of the next customer). The value of the parameter β was varied from 1 to 10, the others parameters were fixed: $\alpha = 1, p = 0.2, O = 524$. Table 2 shows that the value of the parameter β virtually doesn't influence on the solution, the best solution is reached with the values of parameters are equal to 3 and 5.

Table 2

Influence of the parameter β on the quality of the solution *L*

β	1	3	5	7	9
<i>L</i>	583.4	561.1	561.4	562.2	587.0

Table 3 shows some results for different combinations of α and β .

Table 4 shows the best solutions, obtained for 3 benchmark problems.

The behaviour of the parameters α and β is the same as in the first benchmark problem.

Table 3

Influence of the combinations of the parameters α and β on the quality of the solution L

α	β	L	α	β	L
2	3	561.1	2	12	575.5
2	4	601.3	3	5	601.1
2	5	602.5	3	10	563.7
2	8	573.7	3	12	601.7
2	10	566.4	3	15	602.1

Table 4

The best obtained solutions

The problem	The best known solution	The best obtained solution
pr01	524.6	559.9
pr02	835.3	952.2
Pr03	826.1	962.1

6. CONCLUSIONS

The results of the computational experiment showed that the selection of the parameters α and β combinations influences on the quality of the obtained solution of the SDVRP: if α is large and β is small the obtained solution can be too worse as the best known solutions. The obtained results show that the best solutions are reached with β 3–5 times larger than α .

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