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BILEVEL OPTIMIZATION PROBLEM: THE MODEL AND ITS PROPERTIES

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Abstract. The bilevel optimization problem is formulated and possible transformations into problems with explicit or implicit constraints are given. These are nondifferentiable and nonconvex optimization problems or those with generalized equations. Optimality conditions, properties and possible approaches to solve these problems are topic of the talk.

Keywords: Bilevel optimization; nondifferentiable and nonconvex optimization; optimality conditions.

The bilevel optimization problem can be modelled as

$$min\, F(x,y) s.t.\, G(x) \leq 0, (x,y) \in grp\, S,$$

where S denotes the set of optimal solutions of a second, parametric optimization problem

$$S(x) = min\{ f(x,y) : g(x,y) \leq 0 \}$$

Here, all functions F, G_i, f, g_i are assumed to be sufficiently smooth, f, g_i are convex with respect to y. While the first problem is the bilevel or upper level problem, the second one is called the lower level problem. We assume that minimization in the upper level problem is taken with respect to both variables x, y.

To transform this problem into one with explicit or implicit constraints there are at least three possibilities:

1. Since the lower level problem is a convex optimization problem, the Karush–Kuhn–Tucker constraints are necessary and sufficient optimality conditions provided a regularity condition as Slater's conditions is satisfied. If these conditions are used to replace the lower level problem, a so-called mathematical program with equilibrium constraints (MPEC) arises. This problem is most often used to replace the optimization problem. Unfortunately, both problems are equivalent only if global optima are searched for. A local optimal solution of the MPEC is in general not related to a local optimal solution of the bilevel problem.

2. The optimal function value of the lower level problem can be used to transform the bilevel problem is one with (in general nonsmooth) inequality constraints. The resulting problem is equivalent to the bilevel problem even if local optimal solutions are used. But, again generally used constraint qualifications as the (nonsmooth) Mangasarian–Fromovitz constraint qualification is violated at every feasible point.

3. The necessary and sufficient optimality conditions for the lower level problem can be formulated as generalized equation. The resulting problem is again fully equivalent to the bilevel problem.

These three transformations can be used to formulate optimality conditions for the bilevel programming problem using different constraint qualifications, as e. g. (partial) calmness. The MPEC has been investigated deeply in literature. The C- and M-stationarity conditions for MPEC's are adapted to the bilevel problem, see e. g. [1]. To compute B-stationary solutions for the bilevel programming problem, e. g. the elastic mode SQP algorithm for MPEC's [2] can be used.

Those and related results (for the other transformations) will be presented.

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