# IRREGULARITY ESTIMATION OF PLANAR STRUCTURES BASED ON COLOUR MIXING 

R. Chirikov ${ }^{1}$, P. Rocca ${ }^{2}$, E. Grakhova ${ }^{3}$<br>${ }^{1}$ chirikov.ru@gmail.com, ${ }^{2}$ paolo.rocca@disi.unitn.it, ${ }^{3}$ eorlingsbest@mail.ru<br>${ }^{1,3}$ Ufa State Aviation Technical University, Russia<br>${ }^{2}$ University of Trento, Italy

Submitted 2014, June 10


#### Abstract

This paper presents a new approach to the estimation of the irregularity of planar rectangular structures. The method is based on the colour mixing principle. Although the developed method can be applied to structures filled with any shapes of objects, in this work we consider structures tiled with polyominoes. Some analysis is shown that demonstrates efficiency of the method.


Key words: irregularity; polyomino; colour mixing; planar structures.

The bin packing problem has evolved from a purely theoretical curiosity into application oriented research activities during tens of years of studies. The problem is flexible in its formulation and therefore has many different variations according to the application scenarios. It has one-, two-, three- and multidimensional versions. Another two important characteristics of the problem are shapes of the objects to be placed and shape of the container.

One important branch of the bin packing problem is packing polyomino-shaped objects into a two-dimensional rectangular area [1]. Application fields begin with solving the Tetris puzzles and come up to freight placement and electronics design. There can be several shapes of polyominoes that may be rotated. Often the shapes can be even flipped producing eight different orientations of a single polyomino.

Recently the studies of irregular subarraying of large planar phased antenna arrays began [2]. The reason for using irregular subarrays is to suppress the sidelobe level in the radiation pattern of an array by breaking the periodicity of the structure. In other words, subarrays of irregular shapes are supposed to provide highly irregular structures of the antenna arrays. Since all the elements in antenna array are equal it is logical to use the polyominoshaped subarrays [3].

Each particular structure of an antenna array produces unique radiation pattern. Research activities on the optimization of the structures have been carried out in the past few years [4]. Those activities use genetic algorithm as the optimization tool. The sidelobe level is simulated by the software. Due to big number of simulations needed and long
time that it takes to simulate one structure it would be useful to eliminate the simulation of sidelobe level and focus instead on obtaining a highly irregular structure.

In this work we present one approach to estimate the irregularity of structures tiled with polyominoes. This approach has been implemented and corresponding results are provided.

## MATHEMATICAL FORMULATION

For obvious reasons it is impossible to run hardware experiments during the optimization of an antenna array. There are software libraries for numerical simulations of the sidelobe level. Depending on the sizes of the array and accuracy the simulation may take from half a second up to several minutes. This time multiplied by the number of iterations of the genetic algorithm and population size grows to hours spent on one experiment. In order to solve this problem a task was set to find another optimization criterion that could replace the sidelobe level and be quicker to calculate.

In works of Mailloux [2] it is stated that the sidelobe suppression is proportional to the irregularity of the array structure tiled with subarrays. Therefore, we should search for a criterion that could estimate the irregularity of a structure.

Irregularity of a structure tiled with polyominoes may mean that it does not have patterns repeated with some spatial periodicity. At the same time a pattern can be represented by a single polyomino as well as a group of two, three or more. So, it is important to consider uniform distribution of not only all eight orientations of the polyomino,
but also groups of such polyominoes. For this purpose it was proposed to use the principle of colour filtering. Its essence is in the following.

According to the RGB model, all the colours can be obtained by mixing three basic colours: red, green and blue (Fig. 1). The basic colours are orthogonal to each other: they cannot be obtained by mixing two other colours.


Fig. 1. RGB model: three basic and three secondary colours

If we mix each pair of basic colours in equal proportion we will get three secondary colours:

$$
\begin{aligned}
& \text { red + green = yellow; } \\
& \text { red + blue = magenta; } \\
& \text { green }+ \text { blue = cyan } .
\end{aligned}
$$

In total we can use these six colours. We paint polyominoes in the structure with these colours. Each colour is associated with one orientation. In Fig. 2 a structure is shown where all the polyominoes are painted in their colours.

Now let us describe colour channels. Colour channels correspond to the basic colours of the model. In RGB it is red, green and blue. They say a colour is visible in a channel if the corresponding basic colour is used to obtain it. Therefore, in the red channel among our six colours we will see red, yellow and magenta. In the green channel it is green, yellow and cyan while in blue channel blue, cyan and magenta.

Now, if we "turn on" only one channel we will see only those polyominoes in the structure that are painted in the corresponding visible colours. Fig. 3 shows the initial structure in each of three channels.

But we ought to remember that every polyomino in the structure has eight orientations, while there are only six basic and secondary colours. Black colour is used to designate invisible polyominoes and holes that are invisible in any channel. White colour is useless because it is visible in all channels.


Fig. 2. Example of a structure in which polyominoes are painted in colours according to orientations

It is impossible to find four orthogonal colours. But we can abstract our mind from colours and transfer the same principle (mixing and elicitation) to other objects. In this work the prime numbers have been chosen as such objects. Four imaginary colours act as basic: $C_{2}, C_{3}, C_{5}$ и $C_{7}$. They are orthogonal and they don't divide by one another. Their multiplication will represent mixing. Since the numbers are prime, every product will be divisible by only the numbers that represent basic colours. In total there are six secondary colours:

$$
\begin{align*}
& C_{2}+C_{3}=C_{6} ; \\
& C_{2}+C_{5}=C_{10} ; \\
& C_{2}+C_{7}=C_{14} ;  \tag{1}\\
& C_{3}+C_{5}=C_{15} ; \\
& C_{3}+C_{7}=C_{21} ; \\
& C_{5}+C_{7}=C_{35} .
\end{align*}
$$

We will use only two basic colours ( $C_{2}$ and $C_{3}$ ) and six secondary to paint eight orientations of polyominoes.

Then we need somehow to estimate the uniformity of visible elements in the structure in each channel. For that we calculate the number of visible elements in all the rows and columns and their standard deviation (separately among rows and columns). The average value is set exactly to the number of elements in a row/column per one channel. In channels $C_{2}$ and $C_{3}$ four colours are visible, while in channels $C_{5}$ and $C_{7}$ only three. Therefore we divide the number of elements in a row/column by 3.5 to obtain the average:

$a$

$b$

c

Fig. 3. Views of the structure in red (a), green (b) and blue (c) channels

$$
\begin{gather*}
U_{a v g}^{(C)}=N / 3.5  \tag{2}\\
V_{a v g}^{(C)}=M / 3.5 \\
\sigma^{l}=\sqrt{1 / M \sum_{i=1}^{M}\left(V_{i}^{(C)}-V_{a v g}^{(C)}\right)^{2}} \\
\sigma^{-}=\sqrt{1 / N \sum_{i=1}^{N}\left(U_{i}^{(C)}-U_{a v g}^{(C)}\right)^{2}} \tag{3}
\end{gather*}
$$

where $\sigma^{\prime}, \sigma^{-}$- standard deviation of visible elements in a row and column in channel $C, U_{i}^{(C)}$ and $V_{i}^{(C)}$ - number of visible elements in the $i$-th row or column in channel $C, M$ and $N$ - number of rows and columns in the structure.

By this the information about the uniformity of the elements distribution for each colour channel is extracted. Then all the standard deviations are summed up forming a numerical value of the irregularity of the structure $R$ :

$$
\begin{equation*}
R=\sum_{C}\left(\sigma^{-}+\sigma^{-}\right) . \tag{4}
\end{equation*}
$$

The optimization criterion in this case will be positive minimization down to zero, meaning uniform distribution of visible elements among rows and columns in all the channels and, therefore, absence of repeated patterns inside the structure.

## EXAMPLES

Here we provide two examples of $32 \times 32$ structures tiled with polyominoes for which the value of irregularity was calculated. The structures were obtained by the Snowball algorithm [5]. It adds margins around the structure during its work and then cuts them off. That is why polyominoes close to borders are not complete. Time needed to obtain the structures is depreciatingly little.

## Example 1: L-trominoes

Input parameters:

- structure size: $M=N=32$;
- polyomino type: L-tromino.

Fig. 4 shows the structure. It is characterized by a big number of small polyominoes and absence of holes. Output data is presented in Table 1.

Table 1
Output of the first example

| Parameter | Value |
| :---: | :---: |
| Number of polyominoes | 363 |
| Number of holes | 0 |
| Fullness of the structure, $\%$ | 100 |
| Irregularity | 346.36 |



Fig. 4. Structure in the first example

## Example 2: L-octominoes

## Input parameters:

- structure size: $M=N=32$;
- polyomino type: L-octomino.

Fig. 5 shows the structure. Polyominoes of larger size decreases the total number that can be put into the structure but also leads to several holes. Output data is presented in Table 2.

Table 2

## Output of the second example

| Parameter | Value |
| :---: | :---: |
| Number of polyominoes | 144 |
| Number of holes | 14 |
| Fullness of the structure, $\%$ | 98.63 |
| Irregularity | 370.54 |

L-shaped trominoes are the simplest irregular polyominoes. In fact, they have only four orientations. Therefore, it is likely that there are going to be repeated small groups of them over the structure. L-shaped octominoes are free from this effect. They have eight different orientations. This explains why the irregularity of the second structure is higher than of the first one.

## CONCLUSION

In this paper presented a new approach to the estimation of irregularity of planar rectangular structures tiled with polyomino-shaped objects. The proposed method is based on the principle of colour mixing and elicitation in the colour channels. The idea of using colours was taken because it is necessary to check how uniformly distributed not only


Fig. 5. Structure in the second example
single polyominoes, but also groups of two, three and more. For the reason that eight colours needed and RGB model provides only six (three basic and three secondary), a shift was done towards imaginary colours represented by prime numbers. Having four "channels" ten orthogonal "colours" have been obtained.

This approach has been implemented into software and run against several structures two of which are presented here. Examples have proven that the math behind the method provides reasonable results. A $32 \times 32$ structure tiled with Loctominoes has shown higher irregularity than the one with L-trominoes.

## REFERENCES

1. B. H. Gwee, M. H. Lim, "Polynominoes tiling by a genetic algorithm," in Computational Optimization and Applications Journal, vol. 6, no. 3, pp. 273-291, 1996.
2. R. J. Mailloux, et al., "Irregular polyomino-shaped subarrays for space-based active arrays," in International Journal of Antennas and Propagation, vol. 2009, 9 p., 2009.
3. P. Rocca, R. Chirikov, and R. J. Mailloux, "Polyomino subarraying through genetic algorithms," in 2012 IEEE International Symposium on Antennas and Propagation (APSURSI), 2012.
4. R. Chirikov, Optimization of complex polyomino-shaped objects placement by a genetic algorithm (applied to antenna array design, (in Russian). Ufa: USATU, 2014.
5. Чириков P. Ю., Rocca R., Багманов В. Х., Султанов А. X. Алгоритм проектирования фазированных антенных решеток для спутниковых систем связи // Вестник УГАТУ. Т. 17, № 4. C. 159-166. [ R. Chirikov, et al., "Algorithm for phased antenna array design for satellite communications,"(in Russian), in: Vestnik UGATU, vol. 17, no. 4 (57), pp. 159-166, 2013.]

## ABOUT AUTHORS

CHIRIKOV, Roman Yurievich, Dipl. Engineer (UGATU, 2010). Cand. of Tech. Sci. (UGATU, 2014). Dr. of Philosophy (UNITN, 2014).

ROCCA, Paolo, Assistant Prof., Dept. of Information Tech. and Comp. Science. Master of Science (UNITN, 2005). Dr. of Philosophy (UNITN, 2008).
GRAKHOVA, Elizaveta Pavlovna, Postgrad. (PhD) Student, Dept. of Telecommunication Systems. Dipl. Engineer (UGATU, 2012).

## МЕТАДАННЫЕ

Название: Оценка неупорядоченности плоских структур на основе смешения цветов.
Авторы: Р. Чириков ${ }^{1}$, П. Рокка ${ }^{2}$, Е. Грахова ${ }^{1}$.

## Организации:

${ }^{1}$ ФГБОУ ВПО «Уфимский государственный авиационный технический университет» (УГАТУ), Россия.
${ }^{2}$ Университет Тренто (UNITN), Италия.
Email: chirikov.ru@gmail.com.
Язык: английский.
Источник: Вестник УГАТУ. 2014. Т. 18, № 5 (66). С. 26-30. ISSN 2225-2789 (Online), ISSN 1992-6502 (Print).

Аннотация: Эта статья представляет собой новый подход к оценке неравномерности плоских прямоугольных структур. Метод основан на принципе смешения цветов. Хотя разработанный метод может быть применен к структурам, заполненным объектами любой формы, в этой работе мы рассмотрим структуры, составленные из полимино. Приведен анализ результатов, демонстрирующий эффективность метода.
Ключевые слова: неупорядоченность; полимино; смешение цветов; плоские структуры.

## Об авторах:

ЧИРИКОВ Роман Юрьевич, дипл. инж. по многоканальным телеком. системам (УГАТУ, 2010). Канд. техн. наук по сист. анализу, упр. и обр. информации (УГАТУ, 2014). PhD по телекоммуникациям (Тренто, 2014). Иссл. в обл. беспроводных систем связи.

РОККА Паоло, доц. каф. инфотехники и информатики университета г. Тренто. М-р телекоммуникаций (Тренто, 2005). PhD по телекоммуникациям (Тренто, 2008). Иссл. в обл. электромагнетики.
ГРАХОВА Елизавета Павловна, асп. каф. телекоммуникационных систем. Дипл. инж. по радиосвязи, радиовещанию и телевидению (УГАТУ, 2012). Иссл. в обл. беспроводных систем связи.

