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CLASSES OF TRAJECTORIES OF PERSECUTION

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Annotation. Classification of laws of persecution motion is offered in the paper. Researching of persecution modes in number of specific cases was performed. Various modes of motion control using GPS/GLONASS are considered.

Key words: management of dynamic motion; persecution trajectories; GPS; GLONASS.

ABSTRACT

Precision of positioning in GPS/GLONASS systems is getting better extending areas of application of global positioning systems data. Improving of positioning accuracy based on enhanced mathematical algorithms shown in our work [1]. It is getting possible to control of motion of carrier. L.S. Pontryagin and coworkers developed effective in practical applications theory of control of dynamic motion [2]. Dynamic motions defined by the system of equation are considering.

$$\frac{dx^i}{dt} = f^i(x^1, \ldots, x^n, u^1, \ldots, u^r, t), i = 1 \ldots n, \quad (1)$$

where $x^1, \ldots, x^n$ are the phase coordinates and velocities of the managed object, $u^1, \ldots, u^r$ are functions managing turn wheel, brakes and accelerators forming the closed limited set U in the r-dimensional space of object control, t is the time [3, 4].

Dynamic movement control problem is to selection the motion control signals $u^i = u^i(t)$ for which the system (1) gives an unique solution $x^i = x_i(t)$, corresponding to the initial data $x_i(t_0) = (x_0)i$, $i = 1, \ldots, n$, which satisfy to our predefined condition of motion. The integral feature usually determines optimality condition of motion

$$J = \int_{t_1}^{t_2} f^{n+1}(x^1, \ldots, x^n, u^1, \ldots, u^r, t)dt\,, \quad (2)$$

which has the property of transitivity for autonomous systems.

In the case where the optimality criterion is a requirement of minimal motion time between two given points, or closed region of the phase space of generalized coordinates and velocities of the managed moving body, the problem of managing becomes a problem of performance according to L. S. Pontryagin.

Performance problems are simplest because optimality function in this case becomes equal to one. These ones are well known and there are number of solutions for specific classes of movement managing problems [2]. L. S. Pontryagin and coworkers found partially continuous managing signals $u(t)$ located on the border of the limited set of values U during all the time are satisfy to solution of performance problems. Therefore, to fastest movement from one point of trajectory to another driver should speed up at one point and have to maximum push on the brake at another point. In many practical situations, that a strategy is a good choice, but every driver knows the driving rule: accelerates smoothly and brake smoothly. It satisfies to another optimality condition of motion. Each optimality condition defines the own class of optimal controls and optimal trajectories. In the area of intersection of classes of managed movement trajectories simultaneously satisfies two or more of the conditions of optimality.

The simplest class of managed motion is a straight linear motion with constant velocity. Managing signals are constant and equal to zero [5].

We choice an optimal motion with minimal integral curvature as a generalized straight linear motion

$$J = \int_{t_1}^{t_1+\tau} \sigma(t)u(t)dt \quad (3)$$

or more common class – the motion with minimal torsion. Trajectories of performance with partially constant acceleration are given by quadratic splines. They cannot always approximate trajectory of persecution on a finite time interval, which have a smooth managing.

The problem of motion managing for a minimum curvature and torsion we call problem of persecution.

Communication between GLONASS/GPS receiver with satellite system is carried out at an interval Δt. However, persecution time may be both larger and smaller Δt.

Determining of the persecution time is a very important problem both in theory of managing and in theory of persecution. If the time between communication sessions is less than the persecution time $\Delta t < \tau$, then each session gives final trajectory persecution correction, time of persecution and managing signals [6].

Anyway, final time of persecution satisfies to own optimal law of persecution $x_i(t)$, usually different from the straight linear motion.

We proffer classification of the available laws of persecution based on geometric and algebraic approaches.

We can solve the generalized two point Cauchy problem for each class of trajectories and find control commands in each class of trajectories for the dynamic laws of persecution based on the selected class of the persecution.

Finally, the computer subsystem initially selects a class of trajectory then finds laws of dynamic control and solves the persecution problem based on the relative positions and velocities of interceptor and target.

In the simplest case, when the target does not deviate from the predetermined path, the command of persecution control is equal to zero. If the target is retarding linearly, the persecution command for increasing of linear tangent velocity is run. There is a finite set of situations that require more complex trajectories of persecution, which are analyzed below.

THE GEOMETRICAL APPROACH

1. CRITERIA OF OPTIMALITY AND EQUATION OF PERSECUTION

Its known geometrical characteristics of trajectory of movement includes unit vectors for accompanying orthoreper: $\frac{d\bar{r}}{ds} = \tau$ – unit vector of velocity tangent to the trajectory $\frac{d^2\bar{r}}{ds^2} = \sigma \cdot n$, n – unit vector of normal to the trajectory, σ – curvature of the trajectory $\left[\frac{d\bar{r}}{ds}, X, \frac{d^2\bar{r}}{ds^2}\right] = \sigma \cdot b$, b – unit vector of binormal of the trajectory, s – length of the trajectory's arc, – a natural parameter; $\sigma_2 = -\sigma^{-2} \cdot \left[\frac{dr}{ds} \times \frac{d^2r}{ds^2}\right] \cdot \left(\frac{d^3r}{ds^3}\right)$ – second curvature or twisting of the trajectory [3]. A straight line has zero curvature $\sigma = 0$ and as a result zero twisting $\sigma_2 = 0$. They forms the first geometrical class of trajectories widely using in dynamic games. The next class of trajectories are the zero torsion trajectories with smallest curvature.

1.1. Plane motion with minimal integral curvature. Let us require functional (3) on the optimal trajectories of persecution should be smallest without using of dynamic equations of managed object (1). Using the time t as a parameter we have following:

$$\dddot{x}(t)\ddot{y}(t)\dot{z}(t) - \dddot{x}(t)\ddot{z}(t)\dot{y}(t) - \dot{x}(t)\dddot{y}(t)\dot{z}(t) + \dot{+}x(t)\dddot{z}(t)\dot{y}(t) + \dot{x}(t)\dddot{y}(t)\ddot{z}(t) - \dot{x}(t)\dddot{z}(t)\ddot{y}(t)=0, \quad (1.1.1)$$

which means the linear dependence of the velocity vectors, acceleration and acceleration of acceleration:

$$a_1(t) \cdot \begin{bmatrix} \dddot{x}(t) \\ \dddot{y}(t) \\ \dddot{z}(t) \end{bmatrix} = a(t) \cdot \begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix} + b(t) \cdot \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} \quad (1.1.2)$$

Equation (1.1.2) represents a general type of linear homogeneous differential equation of the second order relatively components of interceptor's vector of velocity. This one has two fundamental solutions relatively vector of velocity leading to the plane motion. Indeed, the movement will be flat if the twisting of trajectory

$$\sigma_2 = -\sigma^{-2} \cdot \left[\frac{dr}{ds} \times \frac{d^2r}{ds^2}\right] \cdot \left(\frac{d^3r}{ds^3}\right) \quad (1.1.3)$$

will be equal to zero in every point [3]. However, mixed multiplication of vectors in the right part of statement (1.1.3) becomes zero due to linear dependency (1.1.2), as the determinant of a matrix with linearly dependent rows.

In the particular case of $a_1(t) = 0$ the order of equation is decreasing, trajectories becomes straight linear and under $a(t) = 0$ motion becomes inertial. Simplified variants of plane motion are discussed above. In the general case of plane motions we can consider $a(t) = 1$. We can consider a plane passing through the radius-vector connecting interceptor at initial time of persecution and the target at final time of persecution and vector of velocity of interceptor at initial time of persecution as a plane of persecution. Plane of persecution may change when adjusting the persecution.

Arbitrary coefficients of equation connected to the control signals and have to be defined by methods of reversal problems of dynamics. Solutions founded of conjugated equations (1.1.1) or (1.1.2) with some control signals $a_1(t)$, $a(t)$, $b(t)$ we substitute into equation (1), from which we find the control signals $u_k(t)$ controlled dynamical system [7].

1.2. Movement with minimal integral torsion. Planar motion of persecution is quite capable to solve the first problem of persecution and is not able to solve the second problem of persecution if the velocity vector at the time of completion of the persecution has a component orthogonal to the plane of the persecution.

In this case, a wider class of trajectories, such as converging spiral or helix – trajectory with torsion that goes out of the plane of persecution is needed.

To do this, we can choose the limitation in form of the generalization of statement (1.1.1) or the generalization of equation (1.1.2). We may require the minimal of functional

$$S = \int_{t_1}^{t_1+\tau} \left(\sigma 2 + \lambda \left(\frac{d\bar{r}}{ds}\right)^2\right) ds. \qquad (1.2.1)$$

σ_2 here is a second curvature of trajectory defined by formula (1.1.1) [3].

Using the orthogonality properties of the unit vectors of the accompanying benchmark, we can cause the integral (1.2.2) to the form

$$S = \int (b(s) dn(s) + \lambda ds) \qquad (1.2.2)$$

Variation of functional (1.2.3) leads to the complex nonlinear equation interesting in some cases

$$\left(\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3}\right)\left(-\frac{1}{\sigma^2}\right) - -\frac{d}{ds}\left(\left(\frac{d^3\overline{r(s)}}{ds^3} \times \frac{d\overline{r(s)}}{ds}\right)\left(-\frac{1}{\sigma(s)^2}\right)\right) + -\frac{d^2}{ds^2}\left(\left(\frac{d\bar{r}}{ds} \times \frac{d^2\bar{r}}{ds^2}\right)\left(-\frac{1}{\sigma^2}\right)\right) - -\frac{d}{ds}\left(\left(\frac{4}{\sigma^3} \cdot \frac{d^2\bar{r}}{ds^2}\right) \cdot \left(\frac{d\bar{r}}{ds} \times \frac{d^2\bar{r}}{ds^2}\right) \cdot \frac{d^3\bar{r}}{ds^3}\right) + 2\lambda \frac{d\bar{r}}{ds} = 0. \qquad (1.2.3)$$

However, following a geometrical logic, as the conjugated dynamic equations we choose the generalized equation (1.1.3) of planar motion

$$\dddot{r}(t) = a(t)\ddot{r}(t) + b(t)\ddot{r}(t) + c(t)\dot{r}(t), \qquad (1.2.4)$$

which have three fundamental solutions. Two fundamental solutions gives a planar curative motion and the third fundamental solution may give a torsion of trajectory [8].

2. CLASSES OF PLANAR TRAJECTORIES OF PERSECUTION

2.1. Planar circular motion. The purpose of this and subsequent subsections is the solution of targeting problem at a particular class of trajectories. In case of constant speed planar motion the requirement of minimal integral curvature leads to the constant curvature round motion. This class of persecution is the second one after straight-line motion.

Algorithm is following. For a given initial velocity vector $\{V_x, V_y\}$ find the line perpendicular to the initial velocity vector and passing through the initial point $\{X_0, Y_0\}$. For given initial $\{X_0, Y_0\}$ and final $\{X_1, Y_1\}$ points of trajectory of persecution define coordinates of the center of curvature of the trajectory $\{x, y\}$ and the radius of curvature.

$$R = \frac{\sqrt{\frac{\left(X_0^2 - 2X_0X_1 + X_1^2 + Y_1^2 - 2Y_1Y_0 + Y_0^2\right)^2 (V_y^2 + V_x^2)}{(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)^2}}}{2},$$

$$x = = \frac{-V_yX_0^2 + V_yX_1^2 + Y_1^2V_y - 2Y_1V_yY_0 - 2Y_1V_xX_0 + Y_0^2V_y + 2Y_0V_xX_0}{2(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)},$$

$$y = = -\frac{2Y_0V_yX_0 - 2Y_0V_yX_1 - Y_0^2V_x + V_xX_0^2 - 2V_xX_0X_1 + V_xX_1^2 + V_xY_1^2}{2(-V_yX_0 + V_yX_1 - Y_1V_x + Y_0V_x)},$$

$$\varphi = \arctan\left(\frac{V_x}{V_y}\right). \qquad (2.1.1)$$

Formula (2.1.1) solves the Cauchy problem and we can definitely determine the law of prosecution:

$$x(t) = x + R\cos\left(\frac{ut}{R} + \varphi\right),$$
$$y(t) = y + R\cos\left(\frac{ut}{R} + \varphi\right),$$
$$z(t) = 0.$$

The time of persecution $\tau = \frac{2R}{|V_0|} \arcsin\left(\frac{[X_xV_0]}{|X||V_0|}\right)$. Here X – vector from initial to final point of persecution, V_0 – vector of initial velocity.

This class may occur circular trajectories with a constant centripetal acceleration, but with variable velocity and curvature when it is necessary to catch up with the target or close to it with a minimum speed.

One of variant is $R = K \cdot u^2$ with constant K.

The second variant is the twisting spiral.

Initial spiral's slope is defined by vector of velocity at the start of persecution. Indeed let us suppose

$$a(t) = -2\lambda,\ b(t) = -\lambda^2 - \omega^2$$

and find the solution

$$x(t) = a \cdot \exp(-\lambda t) \cdot \cos(\omega t + \lambda),$$
$$y(t) = a \cdot \exp(-\lambda t) \cdot \sin(\omega t + \lambda),$$
$$z(t) = 0. \qquad (2.1.2)$$

Parameter of the low of persecution are defined from initial coordinates and velocities of the target and interceptor.

Note for simplicity we use a relativity principle and we suppose initial point is stationary at the origin. Parameters of the law of persecution are calculated via relative initial coordinates and velocities

$$a = \sqrt{x_0^2 + y_0^2}\ ,$$
$$\lambda = \frac{-vy_0 y_0 - x_0 v x_0}{x_0^2 + y_0^2}\ ,$$
$$\omega = -\frac{v x_0 y_0 - x_0 v y_0}{x_0^2 + y_0^2}\ , \qquad (2.1.3)$$
$$\varphi = \arctan\left(\frac{y_0}{x_0}\right)\ .$$

2.2. Planar trajectories with available management. It has known in the problems of performance control signal take their boundary values from the area of management with an abrupt change [2]. Available control we understand as finite functions belong to the compact set of management *U* [2]. However, in contrast to the results of the theory of performance, our control signals are differentiable functions on the whole of set *U*.

Equation (1.1.3) with substitution $V(t)=\omega(t)\cdot Z(t)$ allows transition to equation

$$\ddot{Z}(t) = f(t) \cdot Z(t), \qquad (2.2.1)$$

where $f(t) = \left(b(t) + \frac{a(t)^2}{4} - \dot{a}(t)\right)$ and function $\omega(t) = e^{\frac{\int a(t)dt}{2}}$, which doesn't consist the first derivative. Among all kinds of functional dependencies the managing signal $a(t)$ in form

$$a(t) = \sum_{k=1}^{N} \frac{a_k}{t - t_k} \qquad (2.2.2)$$

is of particular interest, as it leads to expression

$$\omega(t) = \prod_{k=1}^{N} (t - t_k)^{\frac{a_k}{2}}\ , \qquad (2.2.3)$$

which represent the limited function if all a_k are positive values. The numbers t_k are the moments of time of shifting retarding to accelerating when zeroing relative velocity occur. These moments in time are the moments of finishing of persecution τ. To complain this condition the coefficients of acceleration a_k should be positive and possibly integer values. The equation (2.2.1) have a solution:

$$Z(t) = C_1 \cdot e^{\int \xi(t)dt}, \qquad (2.2.4)$$

where function $\xi(t)$ is found as a solution of equation

$$\frac{d}{dt}\xi(t) + \xi(t)^2 = f(t). \qquad (2.2.5)$$

Now we can limit the force of equation (2.2.1). Note the scale factor "force" $K^2 f(t)$ is transferred to the $\xi(t)$ and t: $t_1 = tK$, $\xi_1 = \frac{\xi}{K}$, that don't appear in the integral (2.2.4) .

2.2.1 The first case. Let $f(t) = 0$. The cause by $b(t) + \frac{a(t)^2}{4} - \dot{a}(t) = 0$. For limb control signals and monotone subject that most closely approaching to Pontrjagin's "relay" type control, put

$$\frac{a(t)^2}{4} - \dot{a}(t) = \frac{n*(n+2)}{(t-\tau)2}, \quad b(t) = -\frac{n*(n+2)}{(t-\tau)2}.$$

In this case, we find that

$$Z(t) = c_1(t + c2),$$
$$V(t) = c_1(t - \tau)^n (t + c_2),$$
$$x_k(t) =$$
$$= \frac{\left(c_{1_k}(t-\tau)^{n+1}((n+1)*t + \tau + c_{2k}\right)}{((n+1)(n+2))} + x_{0k}.$$

Or by beginning coordinates and velocity

$$x_k(t) = \left((x_{1k} - x_{0k})\left(1 + (n+1)\frac{t}{\tau}\right) - V_{0k}t\right)\left(\frac{t}{\tau} - 1\right)^{(n+1)} + x_{1k}. \qquad (2.3.1)$$

The time of persecution have evolved as

$$\tau = \frac{1}{ax}\left(-(n+1)V_{x0} + \right.$$
$$\left. + \sqrt{((n+1)V_{x0})^2 + ax(n+2)(n+1)(x_1 - x_0)}\right). \qquad (2.3.2)$$

2.2.2 The second case. Let $b(t) = 0$. At that cause we have

$$V_k(t) = \frac{(-\tau+t)^n C_{1k}\sqrt{-\frac{\tau-t}{C_{2k}}}(((\frac{-\tau+t}{C_{2k}})^{\sqrt{(n+1)^2-\frac{3}{4}}})^2+1)}{2(\frac{-\tau+t}{C_{2k}})^{\sqrt{(n+1)^2-\frac{3}{4}}}}.$$

Unlike the first case, the relative velocity does not vanish while reaching the time of persecution τ but it becomes the maximum physical and technical conditions for all positive integers even *n*, except for *n* = 0. When *n* = 0, we have uniformly accelerated motion with acceleration.

$$a_m = \frac{C_{1m}}{2C_{2m}} V_k(t) = \frac{C_{1k}(\left(\frac{-\tau+t}{C_{2k}}\right)+1)}{2},$$
$$V_k(\tau) = \frac{C_{1k}}{2}\ ,$$
$$V_k(0) = \frac{(\left(\frac{-\tau}{C_{2k}}\right)+1)}{2}\ ,$$
$$V_k(t) = V_k(0) + a_k \cdot t\ .$$

For arbitrary *n* > 0 motion, as the velocity increases rapidly in the final stage of persecution. This situation can be compared with the slow persecution before the targeting in a straight line and then shooting a laser beam. This one has conditional practical application.

$$x_k(t) =$$

$$= C_{1k}\left(\frac{\left(\frac{t-\tau}{C_{2k}}\right)^{n+\frac{3}{2}+\sqrt{(n+1)^2-\frac{1}{4}}}}{n+\frac{3}{2}+\sqrt{(n+1)^2-\frac{1}{4}}}+ + \frac{\left(\frac{t-\tau}{C_{2k}}\right)^{\frac{3}{2}+n-\sqrt{(n+1)^2-\frac{1}{4}}}}{\frac{3}{2}+n-\sqrt{(n+1)^2-\frac{1}{4}}}\right)+ +x_{0k}.$$

2.2.3. The third case. The control function $b(t)$ such that the controlling force $f(t)$, defined on a finite time interval τ is given by Legendre polynomials

$$P_n(x) = \frac{1}{2^n \cdot n!}\cdot\frac{\partial^n}{\partial x^n}((x^2-1)^n),$$

where $=\frac{t}{\tau}$. Those complex movements can be useful for the persecution of managed aircraft when an effort against the persecution takes place. In simple cases may be useful more simple management techniques, which are defined by special cases of solutions of equation (1.1.3).

2.3. The differentiable case. Let us $b(t) = =\dot{} a(t)$, in equation (3), Then equation (1.1.3) reduces to

$$\dot{U}(t) = a(t)U(t) + \Omega,$$

where $U(t)$ is a vector of velocity and Ω are integration constants. Let us denote

$$u(t) = e^{-\int a(t)dt},$$

then general type solution of the equation becames

$$\begin{bmatrix}\dot{x}(t)\\ \dot{y}(t)\\ \dot{z}(t)\end{bmatrix} = \frac{\begin{bmatrix}\Omega_x\\ \Omega_y\\ \Omega_z\end{bmatrix}\cdot\int u(t)dt + \begin{bmatrix}C_x\\ C_y\\ C_z\end{bmatrix}}{u(t)}. \qquad (2.4.1)$$

Existing of two vectors reduces this solution to the plane case. Solution (2.4.1) in the case of a constant $a(t)$, shows that vector of velocity shifts from one direction to another one exponentially under persecution. Without losing of significance, we can assign

$$a(t) = -\lambda,$$
$$\dot{x}(t) = \frac{\Omega_x}{\lambda} + C_x e^{-\lambda t},$$
$$\dot{y}(t) = \frac{\Omega_y}{\lambda} + C_y e^{-\lambda t},$$
$$\dot{z}(t) = 0.$$

Parameters of the law of motion are express by coordinates and velocities of the initial an final points of persecution and time of persecution:

$$x(t) = V_{x1}t + \frac{V_{x1}-V_{x0}}{\lambda}e^{-\lambda t} + x_0,$$
$$y(t) = V_{y1}t + \frac{V_{y1}-V_{y0}}{\lambda}e^{-\lambda t} + y_0,$$
$$\lambda = -\frac{Vx_0Vy_1 - Vx_1Vy_0}{-Vx_1Y_1 + Vx_1Y_0 + X_1Vy_1 - X_0Vy_1},$$
$$x_0 = \frac{-Vx_1Vy_0X_0 + Vx_1^2Y_1 - Vx_1X_1Vy_1}{Vx_0Vy_1 - Vx_1Vy_0} + + \frac{Vx_1X_0Vy_1 - Vx_1Y_1Vx_0 + Vx_1Y_0Vx_0 + X_1Vx_0Vy_1}{Vx_0Vy_1 - Vx_1Vy_0}.$$
$$y_0 = \frac{Vy_0X_1Vy_1 - Vy_0X_0Vy_1 + Vy_1Vx_1Y_1Vx_1Y_1}{Vx_0Vy_1 - Vx_1Vy_0} + + \frac{-Vy_1Vx_1Y_0 - X_1Vy_1^2 + X_0Vy_1^2 + Y_0Vx_0Vy_1 - Vy_0Vx_1Y_1}{Vx_0Vy_1 - Vx_1Vy_0}. \qquad (2.4.2)$$

Time of persecution is estimated by formula:

$$\tau = -\frac{Vy_0X_1 - Vy_0X_0 + Vx_1Y_1 - Vx_1Y_0}{Vx_0Vy_1 - Vx_1Vy_0} + + \frac{-X_1Vy_1 + X_0Vy_1 - Y_1Vx_0 + Y_0Vx_0}{Vx_0Vy_1 - Vx_1Vy_0}.$$

We have more general case of solution of the equation (1.1.3). Let us transform the equation (3) to the form

$$\frac{d}{dt}\left(\ddot{r}(t) - a(t)\dot{r}(t) + (\dot{a}(t) - b(t))r(t)\right) = = r(t)\frac{d}{dt}(\dot{a}(t) - b(t)).$$

and require $\dot{a}(t) - b(t) = \alpha$ should be constant. Then we have equation

$$\ddot{r}(t) - \alpha(t)\dot{r}(t) + \alpha r(t) = \beta,$$

and it's solution under

$$a(t) = \alpha\cdot t,$$

$$r(t) = = M\left(\frac{a-\alpha}{2a},\frac{3}{2},\frac{at^2}{2}\right)tC_2 + U\left(\frac{a-\alpha}{2a},\frac{3}{2},\frac{at^2}{2}\right)tC_1 + r_0$$

– is a more general type of planar motion. There M and U are special functions.

3. PERSECUTION WITH TORSION

3.1. Screwed spiral. The most important in the theory of the prosecution motion with torsion is a movement along a twisting of spiral.

$$x(t) = R\exp(-\mu\tau)\cos(\omega\tau) - R + x_0,$$
$$y(t) = R\exp(-\mu\tau)\sin(\omega\tau) + y_0,$$
$$z(t) = azt^2 + Vz_0t + z_0. \qquad (3.1.1)$$

Parameters of the law of motion are found from initial and final points of the phase space as following. Initially we have to find radii of curvature R in the iteration process, R initial value is 10. Then we have to fined Z parameter as approximately solution of equation

$$R = \frac{-(x_1-x_0)\cdot\tan(Z) + y_0 - y_1}{\tan(Z)}. \qquad (3.1.2)$$

Then we substitute Z found into the equation for correction of Δ:

$$\Delta = \exp\left(\frac{2ZVx_0}{Vy_0}\right) - \\ - \frac{A}{(-\tan(Z)\,x_1 + \tan(Z)\,x_0 + y_0 - y_1)^2},$$

$$A = 2y_1y_0\tan(Z)^2 - y_0^2\tan(Z)^2 + \tan(Z)^2x_1^2 - \\ -2\tan(Z)^2x_1x_0 + \tan(Z)^2x_0^2 - \\ -y_0^2 + 2y_1y_0 - y_1^2 - y_1^2\tan(Z)^2.$$

Then we do correction $Z = Z + \frac{\Delta}{2000}$ and having a new value of Z we calculate a new correction to (3.1.3) formula

$$az = -\frac{Vz_0 - Vz_1}{2\tau}.$$

Control signals are calculated by formulas:

$$a(t) = -\frac{2t\mu^3 + 3\mu^2 + 2t\omega^2\mu - \omega^2}{t\mu^2 + 2\mu + t\omega^2},$$

$$b(t) = -\frac{(\mu^4 + 2\mu^2\omega^2 + \omega^4)t}{t\mu^2 + 2\mu + t\omega^2}, \qquad (3.1.4)$$

$$c(t) = \frac{\mu^4 + 2\mu^2\omega^2 + \omega^4}{t\mu^2 + 2\mu + t\omega^2}.$$

Where the functions $a(t)$ and $b(t)$ tend to a finite limit, and $c(t)$ – tends to zero. Omitting cumbersome calculations show dependence of the rate of persecution time (Fig. 1).

We can see in this class of trajectories control signals are finite and vary only in the start of persecution until by torsion do not go on the plane prosecution (Fig. 2 and 3).

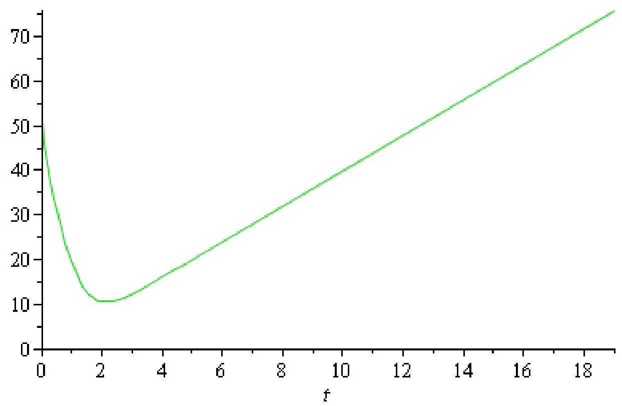

Fig 1. Dependence of velocity of persecution on the time

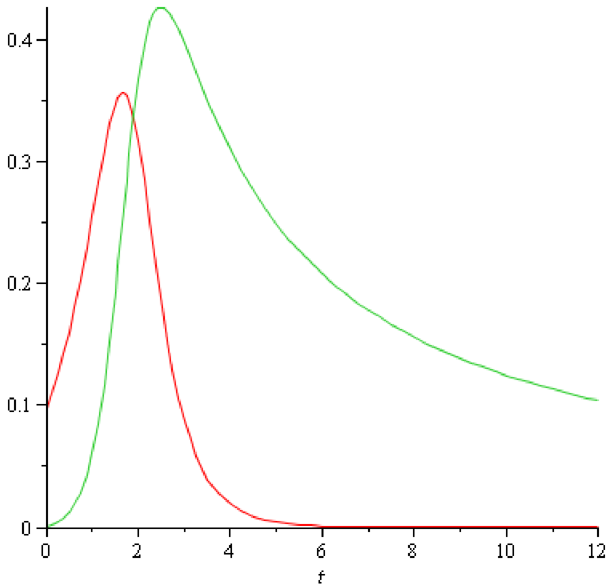

Fig. 2. Dependence of the torsion and curvature of the trajectory on the time of persecution: red line is torsion, green line is curvature

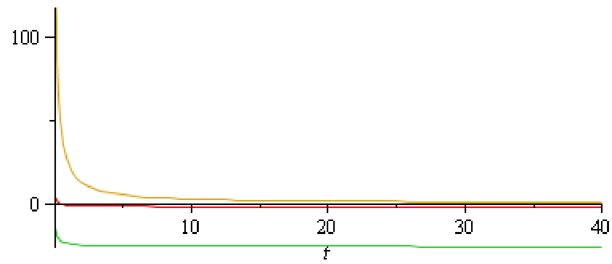

Fig. 3. Dependence of controlling forces on time

4. TRAJECTORIES OF PERSECUTION UNDER GLONASS CONTROL

4.1. Slowly moving objects. We divide the problem into two classes of objects: slowly moving bodies and fast moving bodies to select of trajectories of persecution. In case of slowly moving bodies deviations between real positioning and components of velocity vectors and their tabular values are small within GLONASS sessions receiving time Δt. Ships, cars, people, animals, birds etc. are slowly moving bodies. In case of slowly moving bodies, we can choice the quadratic law of motion – motion with constant acceleration. In the next session of communication, the acceleration may be abruptly slightly changed depending on value and direction of deviations. Let us assign

$$x = a_0 + a_1t + a_2t^2, y = b_0 + b_1t + b_2t^2 \qquad (4.1.1)$$

For which equation we have

$$z = c_0 + \frac{1}{2}c_2t^2 + c_1t.$$

The law of motion (4.1.1) has nine arbitrary constants, which we have to determine based on initial values. Let us suppose coordinates, velocities and accelerations q_x, q_y, q_z, V_x, V_y, V_z, a_x, a_y, a_z were calculated at the moment of previous session t_1.

$$q_x(t) = q_x + Vx_1 + \frac{1}{2}axt^2,$$
$$q_y(t) = q_y + Vy_1 + \frac{1}{2}ayt^2,$$
$$q_z(t) = q_z + Vz_1 + \frac{1}{2}azt^2. \quad (4.1.2)$$

While in this interval of time, the body should move under the law

$$Q_x(t) = Q_x + Ux \cdot t + \frac{1}{2}wx \cdot t^2,$$
$$Q_y(t) = Q_y + Uy \cdot t + \frac{1}{2}wyt^2,$$
$$Q_z(t) = Q_z + Uz \cdot t + \frac{1}{2}wzt^2. \quad (4.1.3)$$

According to condition of persecution under $t=0$, $a_0=q_x$, $b_0=q_y$, $z_0=q_z$. At the moment τ according to condition of persecution (1) and (2) we have equations:

$$\left\{ \begin{array}{r} a_1 + 2a_2\tau = U_x + wx\tau, x_0 + a_1\tau + a_2\tau^2 = \\ = Q_x + U_x\tau + \frac{1}{2}wx\tau^2 \end{array} \right\}$$

from which we find

$$a_1 = \frac{-2x_0+2Q_x+U_x\tau}{\tau},\ a_2 = \frac{-2x_0-wx\tau^2+2Q_x}{2\tau^2}. \quad (4.1.4)$$

Analogous,

$$b_1 = \frac{-2y_0 + 2Q_y + U_y\tau}{\tau},$$
$$b_2 = \frac{-2y_0 - wy\tau^2 + 2Q_y}{2\tau^2},$$
$$c[1] = \frac{-2z_0 + 2Q_z + U_z\tau}{\tau},$$
$$c[2] = -\frac{-2z_0-wz\tau^2+2Q_z}{\tau^2}. \quad (4.1.5)$$

Thus, using formulas (4.1.1) with coefficients (4.1.4) and (4.1.5) we can solve the problem of slowly moving bodies persecution. In this case, GPS/GLONASS receiver should accept current values of position, velocity and acceleration.

4.2. Fast moving objects. In the case of fast moving objects defining of quadratic on time law of motion is not enough. We should take into consideration variations of acceleration approximated by polynomials of third and fourth power. However in case of free of torsion planar trajectories we have relatively complicated third power polynomials analytical expressions making solving of targeting problem too hard. Indeed, in case of free of torsion planar motion with minimal curvature we found

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3,$$
$$y(t) = b_0 + b_1t + b_2t^2 + b_3t^3,$$
$$z(t) = z_0 + C_1\theta(t-\tau)y(t) + $$
$$+C_2\theta(t-\tau)\int e^{F(t)}dt, \quad (4.1.5)$$

where

$$\begin{aligned} F = &-2W_4 - \\ &-2\frac{9a_3^2b_3b_1 - 6a_3^2b_2^2 + 6a_3b_3a_2b_2 - 9a_3b_3^2a_1}{M_1}W_5 - \\ &-2\frac{-9a_3^2b_3b_1 + 9a_3b_3^2a_1 + 6a_3^2b_2^2 - 6a_3b_3a_2b_2}{M_1}W_2 - \\ &-2\frac{1}{M_1}\begin{pmatrix} -9a_3^3b_1^2 - 18a_3^2b_3a_2b_1 + \\ +18a_3^2b_3a_1b_2 - 18a_3a_1b_3^2a_2 + \\ +12a_3a_2^2b_2b_3 + 18a_3^3b_1b_2 + \\ 18a_3^2b_1b_3a_1 - 12a_3a_2^2b_1b_3 - \\ 12a_3^2a_1b_2^2 - 12a_3^2a_2b_2^2 - \\ -9a_3a_1^2b_3^2 + 12a_3a_1b_2b_3a_2 \end{pmatrix}M_3W_3 - \\ &-6\frac{1}{M_1}(-6a_3b_3^2a_1b_1 + 3a_1^2b_3^3 - \\ &-4a_2^2b_2^3 - 4a_3b_3a_2b_1b_2 - \\ &-4a_1b_2a_2b_3^2 + 3a_3^2b_3b_1^2 + \\ &+18a_3b_3^3a_1 - 18a_3^2b_3^2b_1 + \\ &+12a_3^2b_2^2b_3 + 12b_3^3a_1a_2 - \\ &-8b_3^2a_2^2b_2 + 4b_3^2a_2^2b_1 - \\ &-12b_2a_3a_2b_3^2 + 4a_1b_2^2a_3b_3 + \\ &+12a_3b_3a_2b_2^2 - 12a_3b_3^2a_1b_2)M_2W_1. \end{aligned} \quad (4.1.6)$$

W values are a functional dependencies, M values are numerical expressions of $a[k]$, $b[k]$ constants.

$$\begin{aligned} M_1 = {}& 3a_1^2b_3^2 + 36a_3a_1b_3^2 + 12a_1b_3^2a_2 - \\ &-12a_3b_3a_1b_2 - 6a_3b_1b_3a_2 - 4a_1b_2b_3a_2 + \\ &+4a_3a_1b_2^2 - 8a_2^2b_2b_3 + 4a_2^2b_1b_3 + \\ &+12a_3b_3a_2b_1 - 24a_3b_3a_2b_2 + 12a_3^2b_2^2 + \\ &+3a_3^2b_1^2 - 12a_3^2b_1b_2 + 8a_3a_2b_2^2 - 4a_3b_2a_2b_1, \end{aligned}$$
$$M_2 = \sqrt{3b_1b_3 - b_2^2 - 9b_3^2}^{\,-1},$$
$$M_3 = \sqrt{3a_1a_3 - a_2^2}^{\,-1},$$
$$W_1 = \arctan(M_2(b_2 + 3b_3t + 3b_3)),$$
$$W_2 = \ln(a_1 + 2a_2t + 3a_3t^2),$$
$$W_3 = \arctan(M_3(a_2 + 3a_3t)),$$
$$W_4 = \ln(b_1 + 2b_2t + 3b_3t^2),$$
$$W_5 = \ln(b_1 + 2b_2t + 3b_3t^2 - 2b_2 - 6b_3t).$$

Integral parameter in formula (4.1.5) is specific for motion with variable acceleration. M_1 value at denominator of formula becomes equal to zero under zero values a_3 and b_3, which are responsible for variable acceleration. Solutions of the integral are elementary functions, thus we can find twelve constants a_k, b_k using algebraic sums. The simplest maneuver of acceleration allow perform transition from one predefined trajectory to another one within a finite interval of time. In this case, both initial positions and velocities and final ones are coincide. Using these parameters, we can find these twelve constants of the law of motion with above-mentioned formulas. The problem of management of fast moving objects using GPS/GLONASS is that positioning is performing based on quasi range from receiver to constellation within cone of visibility limited by angle of 5–10 degrees above horizon. Lower angle rays cause multipath interference, which decreases accuracy of positioning.

However in case of fast moving objects the acquiring of satellite signals will occur within the sequence of points $M_i(x(t_i), y(t_i), z(t_i))$ not in the one point of trajectory. We find the coordinates and velocities related to the last point along the path defined by the law of persecution $\{x(t), y(t), z(t)\}$. We correct pseudo ranges of previous points with cosine theorem. Thus, we eliminate dilution of precision of positioning of fast moving objects, which may exceed hundreds of meters. In our previous work, we show more precise method of positioning using analytical approach [1]. The approach allow find more accurate trajectories of persecution between communication sessions. Thus, the problem becomes interconnected and under certain conditions, to be special studies it is becomes convergent to the exact limit.

5. ALGEBRAIC CLASSIFICATION OF TRAJECTORIES OF PERSECUTION

Geometrical classification of trajectories of persecution is based on equations describing trajectories of persecution as lines that bend elastic thread with appropriate boundary conditions. There is alternative. We can consider surfaces emerging under deformation of the elastic film and the congruence of geodesic lines on curved surfaces such as the classes of trajectories persecution. In addition, as the world lines on these surfaces, which define the torsion of the Lorentz force. It is known that the equations of Einstein's gravity describe four-dimensional space-time continuum as pulsating elastic film in the space of higher dimension. Therefore, we consider the equation of motion in spaces of general relativity in the form of the Hamilton-Jacobi equation, as a model equation. It is known algebraic classification of these equations based on the algebra of first integrals of motion. This method can be generalized for other Hamiltonian dynamical systems, specifically for the Pontryagin's Hamilton in modern theory of management of dynamic motion.

5.1. Classification of solutions of Hamilton-Jacobi equation

Let us consider the Hamilton-Jacobi equation of test charged particle in general relativity theory as a model equation. The arbitrary gravity field in conjunction with electromagnetic field are able to form various combinations of curvature and torsion of tested charged particles trajectories. It is known the equation of Hamilton–Jacobi in general theory of relativity may be presented as following [4].

$$g_{ik}(x)\left(\frac{\partial}{\partial x_k}S - qA_k\right)\left(\frac{\partial}{\partial x_i}S - qA_i\right) = m^2, \quad (5.1.1)$$

where x_i are generalized coordinates, $g_{ik}(x)$ are metric coefficients expressed as coordinate functions, S is function of action, $p[k] = \frac{\partial}{\partial x_k}S(x, x[0])$ is generalized momentum, $qA_k(x)$ are charge and vector - potential of the electromagnetic field. It is known also first integrals $\sigma_a=\sigma_a(x, p)$ of Hamilton's system form the Lie algebra relatively Jacobi brackets.

$$\{\sigma_a, \sigma_b\} = \frac{\partial}{\partial p_i}\sigma_a \frac{\partial}{\partial x_i}\sigma_b - \frac{\partial}{\partial p_i}\sigma_b \frac{\partial}{\partial x_i}\sigma_a.$$

First integral commutator can be expressed linearly from integrals of algebra with structural constants of algebra $C^r_{a,b}$ which are anti symmetric on lower indexes and satisfy to the Jacobi identity.

$$\{\sigma_a, \sigma_b\} = \sum_{r=1}^{n} C^r_{a,b}\sigma_r \,, \quad (5.1.2)$$

$$\{\sigma_c, \{\sigma_a, \sigma_b\}\} + \{\sigma_a, \{\sigma_b, \sigma_c\}\} + \{\sigma_b, \{\sigma_a, \sigma_c\}\} = 0. \quad (5.1.3)$$

Hamiltonian

$$H(P, x) = g_{ik}\left(\frac{\partial}{\partial x_k}S - qA_k\right)\left(\frac{\partial}{\partial x_i}S - qA_i\right)$$

satisfy to Jacobi brackets because it is integral of motion also.

$$\{H(P, x), \sigma_a(p, x)\} = 0 \,. \quad (5.1.4)$$

Algorithm of composing of classes of trajectories is following.

1. The system of homogeneous quadratic equations relatively unknown components of algebra (5.1.3) structural constants containing n linear first integrals of motion is solving.

$$\sigma_a = J_a(x)^i p_i,\ a = 1\ldots n,\ i = 1\ldots 4,\ n = 1\ldots 10 \,. \quad (5.1.5)$$

Based on the found coefficients structural constants $C^r_{a,b}$ excepting isomorphic cases are composing.

2. For each algebra defined by structural constants $C^r_{a,b}$ the system (5.1.2) is solving and first integrals $\sigma_a = J_a(x)^i p_i$ or Kiling's vectors $J_a(x)^i$, $a = 1\ldots n$ are fining.

3. System of equations (2.1.4), which decomposes for linear integrals (2.1.5) into a system of

equations for the components of the Killing metric $g_{ik}(x)$ is solving:

$$D_i J_k(x) + D_k J_i(x) = 0 \ , \qquad (5.1.6)$$

where D_i denotes the covariant derivative with respect to the desired metric $J_k(x) = g_{ik}(x) J_a(x)^i$ and to the Kiling equation for the vector component of electromagnetic field:

$$J_a(x)^i \frac{\partial}{\partial x_i}\left(A_j(x) g(x)^{jm}\right) - \\ -A_j(x) g(x)^{ji} \frac{\partial}{\partial x_i}\left(J_a(x)^m\right) = 0 \qquad (5.1.7)$$

Solving systems of linear partial differential equations (5.1.6) and (5.1.7) we restore the power functions of the each algebraic class with more or less certainty, depending on the number of integrals of algebra n, and with them, and Hamiltonian.

4. As a result, availability of composing of algebraic classes of trajectories is getting possible. There are two approaches. The first one is a direct method. Coordinates on the deformed film, we transfer on locally planar space. Because analytical expression of the law of motion of a certain algebraic class is desirable. To find the law of motion we compose an equation of geodetic line – trajectory without torsion or we compose an equation of world line in present of electromagnetic field – trajectory with torsion not transferring the body from surface of elastic space. In the second case, we solve the problem of embedding a deformed film in the planar space of higher dimension and define a class of trajectories in planar space.

5. Thus, it is possible to solve the problem of persecution in this algebraic class of trajectories. To do this, we solve the Cauchy problem of the transformed equation (5.1.8) at the start and final data of trajectory of persecution. A number of authors solved that hard problem. Levi–Chivita found structural constants of a three membered Lie algebra [5]. A. Z. Petrov and V. R. Caigorodov at Kazan University, Russia found all remaining algebraic structures with real structural constants until tenth order. These authors found canonic forms of Kiling vectors respecting to linear first integrals and satisfying those metrics [6]. One of author of this researching found all electromagnetic fields on Kiling–Petrov–Kaigorodov's vectors [7]. For commutative algebras of first integrals, the problem was solved for classical system by Jarov–Jarovoi and for general case of general relativity was solved by Jarov–Jarovoi's method [8].

Using Maple environment for computer aided mathematical simulation the solution of the problem is possible in general form during a short time not only in canonical Petrov–Kaigorodoc-Zakharov's form. Authors have a number of cases proving the method. Let us consider a case of filth order algebra.

A metric tensor

$$g_{ij} = \begin{bmatrix} -1 & 0 & 0\ 0 \\ 0 & -1 - x^2 & -x 0 \\ 0 & -x & -1 0 \\ 0 & 0 & 0\ 1 \end{bmatrix}$$

satisfy algebra of integrals

$$\sigma_1 = p_1 - y p_2 \ ,$$
$$\sigma_2 = p_2 \ ,$$
$$\sigma_3 = p_3 \ ,$$
$$\sigma_4 = p_4 \ ,$$
$$\sigma_5 = -y p_2 + x p_1 + \frac{y^2 - x^2}{2} p_3 \ .$$

Geodetic lines equations

$$\begin{cases} x'''^{(t)} - x(t) y'^{(t)^2} - y'^{(t)} z'^{(t)} = 0, \\ y''(t) + x(t) x'(t) y'(t) + y'(t) z'(t) = 0, \\ z''(t) + 2\left(-\frac{x(t)^2}{2} + \frac{1}{2}\right) x'(t) y'(t) - x(t) x'(t) z'(t) = 0, \\ \tau''(t) = 0 \end{cases}$$

have solutions:

1. Rectilinear uniform motion along the axis *ox* or axis *oz*;
2. Uniform motion along the axis *oy* and uniformly accelerated motion along the axis *oz*;
3. Uniform motion along the axis *ox* and *oy*, uniformly accelerated along the axis *oz*.
4. The variant that met in Part 1:

$$y(t) = -\frac{C_3^2}{2 e^{\frac{t}{C_3}} e^{\frac{C_5}{C_3}}} - \frac{C_3^2 e^{\frac{t}{C_3}} e^{\frac{C_5}{C_3}} C_4}{2} + y_0 \ ,$$
$$x(t) = \int \frac{\sqrt{-\dddot{y}(t) \dot{y}(t)} \ddot{y}(t)}{\dddot{y}(t)} dt + x_0 \ ,$$
$$z(t) = \int -\frac{\ddot{y}(t) + x(t)\dot{x}(t)\dot{y}(t)}{\dot{x}(t)} dt + z_0 \ .$$

5.2 Algebraic classification of trajectories of persecution by Pontryagin

It is known the Pontryagin's theory of management motion is reduced to the Hamilton form [2].

$$H(\psi, x, u) = \sum_{a=1}^{n+1} \psi_a f(x,u)^a \ . \qquad (5.2.1)$$

$$\frac{\partial x^i}{\partial t} = \frac{\partial}{\partial \psi_i} H, \ \frac{\partial \Psi_i}{\partial t} = -\frac{\partial}{\partial x_i} H \ . \qquad (5.2.2)$$

Moreover, generalized momentum ψ_i is a gradient of some function of action, which leads to the form of the Hamiltonian system of Hamilton - Jacobi equations [2]. Let us transfer the general relativity algorithm of algebraic classification of Hamilton–Jacobi equations to the Hamiltonian system of controlled motion (5.2.1), (5.2.2). In case of planar motion, we have four-dimensional space of variables x_i – two generalized coordinates and two generalized velocities. For this case, structural constants of Lie algebra and respecting first integrals are transferring without changing from researching of Petrov and Caigorodov [4]. Kiling type equations for the Hamiltonian (5.2.1) of autonomous system with r-membered Lie algebra of first integrals

$$\sigma_a = J_a(x)^i \psi_i,\ a = 1\ldots r,\ i = 1\ldots n{+}1.$$

can be presented as

$$\sum_{j=1}^{n+1} J_a(x)^j \frac{\partial}{\partial x^j} f^i(x,u) - \sum_{j=1}^{n+1} \frac{\partial}{\partial x^j}\left(J_a(x)^i\right) f^j(x,u) = 0,$$
$$i,j = 1 \ldots (n+1), a = 1 \ldots r. \tag{5.2.3}$$

Killing vectors belonging to the Lie algebra ensures consistency of the system (5.2.3) and the existence of solutions for the required power functions $f^j(x,u)$, which becomes functions of coordinates of extended phase space [9]. This situation is typical for autonomous dynamical systems, when the control signals are not functions of time, and the functions of the coordinates, and they have entered the control forces implicitly [2]. We have to find the law of motion of managed object and control signals based on found power functions - right sides of equations (1). For movements in three-dimensional configuration space, we have the six-dimensional phase space and the seven-dimensional extended phase space. As far as authors know algebraic structure these spaces are not currently constructed.

Let us consider for instance the planar motion in xoy plane. Let assign $x=x_1$, $y=x_2$, $x'^{(t)} = x_3$, $y'(t) = x_4$.

We will looking for power forces in form of

$$f_1 = x_3, f_2 = x_4, f_3 = x_4,$$
$$\emptyset(x_5), f_4 = -x_3\emptyset(x_5), f_5 = \emptyset(x_5).$$

Let us suppose that first killing integral $J_1 = x_3$, $J_2 = x_4$, $J_3 = \omega \cdot x_4$, $J_4 = -\omega \cdot x_3$, $J_5 = F_5(x_3, x_5)$ is known. Thus equation (5.2.3) gives equation $\phi(x_5) = 1$. And equations of persecution becomes equations of performance with trajectories

$$x(t) = -\frac{C_3\cos(\omega t) - C_4\sin(\omega t) - C_2\omega}{\omega},$$
$$y(t) = \frac{C_3\sin(\omega t) + C_4\cos(\omega t) + C_1\omega}{\omega}.$$
$$V_x(t) = C_3\sin(\omega t) + C_4\cos(\omega t),$$
$$V_y(t) = C_3\cos(\omega t) - C_4\sin(\omega t),$$
$$\tau = t + C_5.$$

CONCLUSIONS

1. Trajectories of persecution satisfying to the condition of minimal curvature and torsion may differ significantly from piecewise time-quadratic or time-cubic laws of performance.

That may be a decisive factor for the solving of problems of persecution. Fig. 4 show above mentioned.

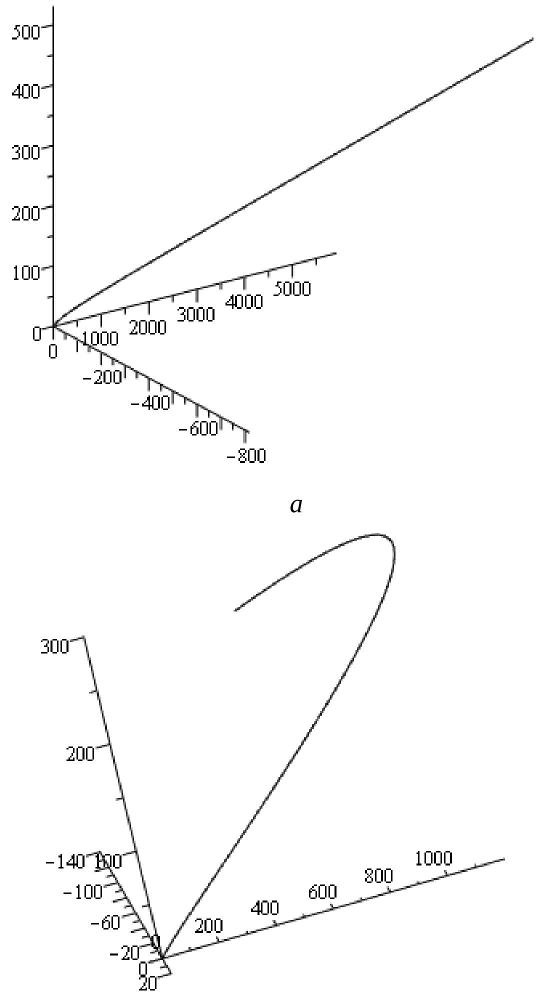

Fig. 4. The trajectory of persecution as a line with minimal curve (a) and the trajectory of persecution as a cubic relationship (b), τ_2 = 6,2033

Initial data in cases of A and B are the same but the final conditions are differ significantly. Curves are coincide at the initial stage and are disperse outside curve of minimum curvature decomposition in power series of Taylor (Fig. 5).

2. We do not need finding the quadratic laws of persecution motion at every small region of persecution because computational load will increase

significantly and both calculation errors and iteration errors will increase extensively. We can use a certain class of laws of motion at the finite trajectory of persecution within a finite time instead. Using this approach, we should do decomposition of trajectories into classes. A geometrical approach containing the set of simplest geometry trajectories could not give us comprehensive and invariant classification of trajectories of persecution. Algebraic classification is better for solving the problem of classification of trajectories.

3. Uniformly accelerated law of motion is using in the theory of performance. Complex trajectory is approximating by splines with appropriate triggering of control signals. In the theory of persecution a set of predefined trajectories is using. Each class of trajectories have own analytical expression and constants as equation parameters. The problem of targeting is solving separately for each class of equations. These solutions include time of persecution and numerical values of trajectory parameters allowing transition from initial point of phase space to the final one. The law of motion with certain numerical parameters allow find control signals as continuous functions of time. Algebraic classification of trajectories of motion is very valuable because it is general and invariant.

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МЕТАДАННЫЕ

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Ключевые слова: управление динамическим движением; траектории преследования; GPS; ГЛОНАСС.

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