

GOVERNMENT CONTROL OF STACKELBERG EQUILIBRIUM AT NATURAL MONOPOLY

E. D. KONOVALOVA¹, A. V. PANYUKOV²

¹konovalova_ekaterina@bk.ru, ²a_panyukov@mail.ru

South Ural State University (SUSU), Russia

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Abstract. Aim of paper is the increasing of effectiveness of government control methodology of natural monopoly. The problem is formulated as two tasks: to build a model of interaction monopolist and buyer at the market; to build a model of interaction government and this market. The market is formalized as non-cooperative non-antagonistic two-person game. Stackelberg equilibrium in mixed strategies is established at the market. Governments influence the equilibrium only non-price methods by setting corrective matrices. The task of finding the optimal government corrective matrices is solved in this paper.

Key words: natural monopoly; adaptability; non-cooperative game; Stakelberg equilibrium; mixed strategy.

1. INTRODUCTION

Government regulation of highly monopolized markets is necessary, but regulation methods are not perfect today. They are one of the most important economic questions at all government levels. The most effective way is to tariff regulation the most effective method is tariff regulation. Tariffs are generated by cost method today, it does not receive excess monopoly profits, but it is not regulated competitive market component.

Regulate of natural monopolies is especially difficult. In developing the regulation methods of these markets is necessary to consider the instability, which is connected with the struggle between the government as a defender of free competitive market, and the government as a guarantor of supplying the population with life-supporting products, which produces a natural monopoly. The government's task is to reduce the impact of monopoly power and at the same time not destroy these companies because they produce high social utility products. This situation demands a dynamic government intervention to balance the interests of monopolies and society [1].

2. THE GENERAL APPROACH TO SOLVING THE PROBLEM

Solution of the problem government adaptation to change market situation is cyclical. On each step government establishes the some market conditions and after that market is seen as a loop system, monopolist and customers interact without government

influence. Government can see extreme market characteristics because it considers loop market. Based on these characteristics government selects new optimal strategy and new matrix of government influence. Then a new step begins.

Two questions are solved for each step:

- to build a model of interaction monopolist and buyer at the specified corrective government control methods (non-price methods);
- to build a model of interaction government and market and estimate the government adaptability characteristics [2].

3. MODEL OF INTERACTION MONOPOLIST AND CUSTOMERS AT THE MARKET

We must consider interaction each customer and monopolist and interaction all customers and monopolist as conflicts. To determine the market situation used game theory methodology. The market is formalized as non-cooperative non-antagonistic bimatrix two-person game (Tabl. 1). Monopolist M and set of customers Π are players of this game [3].

Game process: monopolist (player M) makes the supply and want to maximize profits by increasing the price characteristics of product; customers (player Π) compare the non-price characteristics and price and makes the demand. If player Π demand falls then player M profits also changes [4].

Table 1

Selection of game type

Game type	Imply	In this case
Antagonistic	Opposite aims of players	Monopolist and customers have different but not opposite aims, they have own efficiency criterion
Cooperative	Interaction groups of players	Customers are not coordinating their actions
Many-person game	Many players leads to complicated calculations	All customers have similar characteristics and aims

Let us consider monopolized markets as [2]:

$$\Gamma = (\{M, \Pi\}, \{X_M, X_\Pi\}, \{H_M, H_\Pi\}),$$

with M – monopolist;

$X_M = I = \{1, 2, \dots, n\}$ – set of monopolists strategies, each strategy $i \in X_M$ determines the product S_i and its price P_i ;

Π – set of customers; $X_\Pi = J = \{1, 2, \dots, m\}$ – set of customers strategies, each strategy $j \in X_\Pi$ determines the set of demand D_j and its consumption volumes V_j ;

$H_M = PV^T - \Delta_M$ – monopolist's payoff matrix with elements $H_{Mij} = P_i V_j - \Delta_{Mij}$ – monopolist's profits in case (i, j) , where P_i is monopoly price, V_j is consumption at monopoly price, Δ_M is matrix of government influence for the monopolist;

$H_\Pi = PV^T + \Delta_\Pi$ – customers payoff matrix with elements $H_{\Pi ij}$ that is total utility for consumers given case, matrix Δ_Π of government influence for the customers;

$$\sum_{i \in I, j \in J} \Delta_{Mij} = \sum_{i \in I, j \in J} \Delta_{\Pi ij}.$$

Government influence for the players is not necessarily equal $\Delta_{\Pi ij} \neq \Delta_{Mij}$, but system is closed: government reallocate part of payoff between monopolist and customers.

It should be noted: government influence only non-price methods. It can influence the payoff matrices to selected matrices Δ_M and Δ_Π , but it cannot fix market price and eliminate the monopolist-

customers conflict. Market is two payoff matrices (Fig. 1).

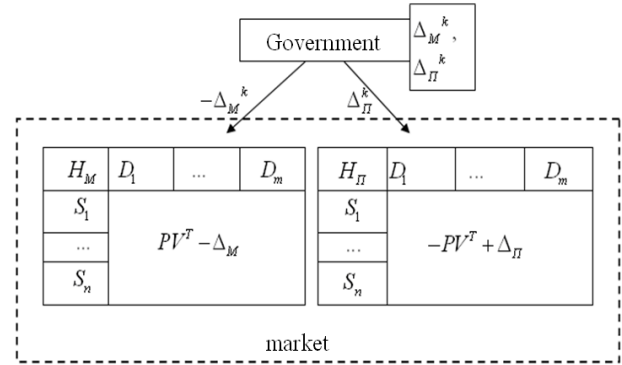


Fig. 1. Formalization of the market [4]

4. THE CONTROL OF EQUILIBRIUM

Stackelberg equilibrium effectively use in monopolized market model. Monopolist is leader, it may declare the price, the customers are driven. Finding the Stackelberg equilibrium in pure strategies, if government excludes deliberately inefficient strategy, is a simple task. Player M knows the payoff functions of both players and chooses a strategy

$x_{iM}^* = \arg \max_{x_i} H_M(x_i, x_j(x_i))$ to maximize

your profit. So $\overline{H_M} = \max_{x_i} H_M(x_i, x_j(x_i))$ is the payoff of player M , which optimally plays as a leader [4].

However, the Stackelberg equilibrium in pure strategies does not reflect proportion of individual consumer opinions in aggregate demand (S) and proportion of each technology in the monopolist technological process (q). Finding the Stackelberg equilibrium in mixed strategies (q^*, s^*) is the closest to the real conditions [2]:

$$\begin{aligned} s^* &= \arg \max_s \left[q^{*T} (-PV^T + \Delta_\Pi) s \right], \\ s(q) &= \arg \max_s \left[q^T (-PV^T + \Delta_\Pi) s \right], \\ q^* &= \arg \max_q \left[q^T (PV^T - \Delta_M) s(q) \right]. \end{aligned}$$

Let k step in the monopoly – customers game at a given government policy (matrices Δ_M^k, Δ_Π^k), equilibrium situation $(q^{(k)}, s^{(k)})$ formed in the market as a result of evolution (i.e. vectors P and V take values $P^{(k)}$ and $V^{(k)}$). This situation can no longer satisfy the government on economic

characteristics or other reasons. In order to establish a new equilibrium (q^{k*}, s^{k*}) , government changes the matrices Δ_M^k and Δ_Π^k to Δ_M^{k*} and Δ_Π^{k*} .

4.1. The control of equilibrium in pure strategies

Find matrices $\Delta_M^{k*}(i^*, j^*)$ and $\Delta_\Pi^{k*}(i^*, j^*)$,

which result all possible equilibrium (i^*, j^*) in pure strategies.

To calculate the corrected payment matrices $H_M = P^{(k)}V^{(k)T} - \Delta_M$ and $H_\Pi = -P^{(k)}V^{(k)T} + \Delta_\Pi$ in which situation (i^*, j^*) is Stackelberg equilibrium in pure strategies, necessary to construct such matrices Δ_M and Δ_Π , which are the solution of linear programming problem [5]:

$$\sum_{i \in I, j \in J} (u_\Pi(i, j) + u_M(i, j)) \rightarrow \min_{u, \Delta} \quad (1)$$

$$(\forall i \in I, j \in J) \quad (-u_\Pi(i, j) \leq \Delta_\Pi(i, j) \leq u_\Pi(i, j), -u_M(i, j) \leq \Delta_M(i, j) \leq u_M(i, j), u_\Pi, u_M \geq 0), (2)$$

$$(\forall i \in I \setminus \{i^*\}, j \in J) \quad (p(i)V(j) - \Delta_M(i, j) \leq p(i^*)V(j) - \Delta_M(i^*, j)), (3)$$

$$(\forall i \in I, j \in J \setminus \{j^*\}) \quad (-p(i)V(j) + \Delta_\Pi(i, j) \leq -p(i)V(j^*) + \Delta_\Pi(i, j^*)), (4)$$

$$\sum_{i \in I, j \in J} \Delta_M(i, j) = \sum_{i \in I, j \in J} \Delta_\Pi(i, j). \quad (5)$$

Government influence which leads to the minimum funds redistribution by the government is rational influence (1). Conditions (3) and (4) reflect the acceptability of strategies i^*, j^* respectively for a monopolist and consumers. Condition (5) reflects a closed system.

Theorem 1. Problem (1)–(5) has an optimal solution [6].

Proof [4]. Set of solutions of (1)–(5) is not empty, it trivial solution is

$(\forall i \in I, j \in J) (\Delta_M(i, j) = \Delta_\Pi(i, j) = p(i)V(j))$ that corresponds to the total funds redistribution by the government.

The problem dual problem (1)–(5) has the form:

$$\sum_{i \in I, j \in J} (t_\Pi(i, j) p_i (v_j - v_{j^*}) + t_M(i, j) v_j (-p_i + p_{i^*})) \rightarrow \max_{r, s, t, u}, \quad (6)$$

$$(\forall j \in J) \quad (-r_M(i^*, j) + s_M(i^*, j) + \sum_{i \in I \setminus \{i^*\}} t_M(i, j) - t_M(i^*, j) + u \leq 0), \quad (7)$$

$$(\forall i \in I \setminus \{i^*\}, j \in J) \quad (r_M(i, j) - s_M(i, j) - t_M(i, j) + u \leq 1), \quad (8)$$

$$(\forall i \in I) \quad (-r_\Pi(i, j) + s_\Pi(i, j) - \sum_{j \in J \setminus \{j^*\}} t_\Pi(i, j) + t_\Pi(i, j^*) - u \leq 0), \quad (9)$$

$$(\forall i \in I, j \in J \setminus \{j^*\}) \quad (r_\Pi(i, j) - s_\Pi(i, j) + t_\Pi(i, j) - u \leq 1), \quad (10)$$

$$r, s, t, u \geq 0. \quad (11)$$

Trivial zero solution $r, s, t, u = 0$ is a feasible solution of the dual problem. Since the problem has a feasible solution of the direct problem and feasible solution to the dual problem, then it has a feasible solution. Theorem 1 is proved.

It should be noted that the problem (1)–(5) has a block structure and so a pair of dual tasks in pure strategies can be solved by decomposition Dantzig-Wolfe. Direct problem is split into two autonomous tasks, and condition (5) is connective task.

The coordinating task:

$$\sum_{i \in I, j \in J} (u_\Pi(i, j) + u_M(i, j)) \rightarrow \min_{u, \Delta}$$

$$\sum_{i \in I, j \in J} \Delta_M(i, j) = \sum_{i \in I, j \in J} \Delta_\Pi(i, j),$$

$$u_\Pi \geq 0, u_M \geq 0.$$

Partial task 1:

$$\sum_{i \in I, j \in J} u_\Pi(i, j) \rightarrow \min_{u, \Delta}$$

$$\left(\begin{array}{l} \forall i \in I, \\ \forall j \in J \end{array} \right) \left\{ \begin{array}{l} \Delta_\Pi(i, j) + u_\Pi(i, j) \geq 0, \\ -\Delta_\Pi(i, j) + u_\Pi(i, j) \geq 0, \\ \Delta_\Pi(i, j^*) - \Delta_\Pi(i, j) \geq \\ \geq p(i)V(j^*) - p(i)V(j). \end{array} \right.$$

Partial task 2:

$$\sum_{i \in I, j \in J} u_M(i, j) \rightarrow \min_{u, \Delta} \left\{ \begin{array}{l} \Delta_M(i, j) + u_M(i, j) \geq 0, \\ -\Delta_M(i, j) + u_M(i, j) \geq 0, \\ \Delta_M(i, j) - \Delta_M(i^*, j) \geq \\ \geq p(i)V(j) - p(i^*)V(j). \end{array} \right. \quad (\forall i \in I, \forall j \in J)$$

The coordinating task is solved by a modified simplex method. To find an initial basic solution is necessary to solve partial tasks 1 and 2.

If u is fixed, the dual problem (6)–(11) reduced to two independent problems of the optimal flow in the limit network. Fix u and consider the dual problem (7)–(8), will move $u = const$ from the right side of inequalities:

$$(\forall j \in J) \left(-r_M(i^*, j) + s_M(i^*, j) + \sum_{i \in I \setminus \{i^*\}} t_M(i, j) - t_M(i^*, j) \leq -u \right), \quad (7')$$

$$(\forall i \in I \setminus \{i^*\}, j \in J) \left(r_M(i, j) - s_M(i, j) - t_M(i, j) \leq 1 - u \right). \quad (8')$$

In (8') is necessary to use the equality

$$r_M(i, j) - s_M(i, j) - t_M(i, j) = 1 - u.$$

For a maximum of (6) variable $t_M(i, j)$ is calculated according to P_i and P_i^* relations:

$$t_M(i, j) = \begin{cases} 0, & \text{if } p_i > p_i^*, \\ 1 - u, & \text{if } p_i < p_i^*, \\ \forall, & \text{if } p_i = p_i^*. \end{cases}$$

Similarly, in the task (9)–(10) variable $t_{\Pi}(i, j)$ is calculated according to v_j и v_j^* relations:

$$t_{\Pi}(i, j) = \begin{cases} 0, & \text{if } v_j > v_j^*, \\ 1 + u, & \text{if } v_j < v_j^*, \\ \forall, & \text{if } v_j = v_j^*. \end{cases}$$

Thus, at a fixed u , the solution of autonomous task blocks (7)–(8) and (9)–(10) is found. If the solutions of partial tasks are not equal, we introduce synthetic variable with surcharge cost to get initial basic solution.

The objective function of the partial task 1 takes the form:

$$\sum_{i \in I, j \in J} u_{\Pi}(i, j) - \pi \sum_{i \in I, j \in J} \Delta_{\Pi}(i, j) \rightarrow \min_{u, \Delta},$$

the objective function of the partial task 2 takes the form:

$$\sum_{i \in I, j \in J} u_M(i, j) - \pi \sum_{i \in I, j \in J} \Delta_M(i, j) \rightarrow \min_{u, \Delta}.$$

Introduce the basic solution to the simplex and after each iteration we check the current solution by optimality conditions:

$$\left\{ \begin{array}{l} \sum_{i \in I, j \in J} u_{\Pi}(i, j) - \pi \sum_{i \in I, j \in J} \Delta_{\Pi}(i, j) - \sigma \geq 0, \\ \sum_{i \in I, j \in J} u_M(i, j) - \pi \sum_{i \in I, j \in J} \Delta_M(i, j) - \delta \geq 0, \end{array} \right.$$

with σ, δ – are simplex multipliers, which respectively corresponding to the limits of partial tasks 1 and 2. If one of the conditions is not satisfied, nonoptimal solution vector of the partial task introduced in the basis.

4.2. The control of equilibrium in mixed strategies

In mixed strategies the optimal (s^*, q^*) and matrices $(\Delta_M, \Delta_{\Pi}) \in D(s^*, q^*)$ are determined by the conditions:

$$\sum_{i \in I, j \in J} (u_{\Pi}(i, j) + u_M(i, j)) \rightarrow \min_{(\Delta_M, \Delta_{\Pi}) \in (s^*, q^*), u} \quad (12)$$

$$(\forall i \in I, j \in J) \left(-u_{\Pi}(i, j) \leq \Delta_{\Pi}(i, j) \leq u_{\Pi}(i, j), -u_M(i, j) \leq \Delta_M(i, j) \leq u_M(i, j), u_{\Pi}, u_M \geq 0 \right), \quad (13)$$

$(\forall q \in Q, s \in S)$

$$(q^T (pV^T - \Delta_M) s \leq q^{*T} (pV^T - \Delta_M) s), \quad (14)$$

$$(q^T (-pV^T + \Delta_{\Pi}) s \leq q^T (-pV^T + \Delta_{\Pi}) s^*), \quad (15)$$

$$Q = \{q \in \mathbf{R}^n : 0 \leq q \leq 1, \|q\|_1 = 1\},$$

$$S = \{s \in \mathbf{R}^m : 0 \leq s \leq 1, \|s\|_1 = 1\}, \quad (16)$$

$$\sum_{i \in I, j \in J} \Delta_M(i, j) = \sum_{i \in I, j \in J} \Delta_{\Pi}(i, j). \quad (17)$$

This problem is a problem of parametric linear programming. Probabilities s and q are analogue values in given limits.

One way of solving this problem is to solve a linear programming problem for each possible sets s , q . But this way makes it necessary to solve an infinite number of linear programming problems. Parametric programming methods solve this problem.

Alternative to parametric programming methods is an imitation discretization s and q according to a predetermined range. Range must be chosen according to a balance of accuracy and computation values. Range selection is a separate non-trivial problem. For each pair of discrete values

(s, q) the matrices $(\Delta_M^{(i^*, j^*)}, \Delta_\Pi^{(i^*, j^*)})$, witch lead to equilibrium in pure strategies $(i^*; j^*)$, are calculated, then calculated their weighted sums.

Proposition 1: Weighted sums of matrices

$$\Delta_M = \sum_{i \in I, j \in J} \Delta_M^{(i,j)} q_i^* s_j^*, \quad \Delta_\Pi = \sum_{i \in I, j \in J} \Delta_\Pi^{(i,j)} q_i^* s_j^*$$

with $(\Delta_M^{(i,j)}, \Delta_\Pi^{(i,j)})$ – solution of the problem (1)–(5) for the equilibrium situation $(i; j)$, are the optimal solution of the problem (12)–(17).

5. CONCLUSION

- Government control of Stackelberg equilibrium at natural monopoly is a part of the research on the problem performance analysis of government control methods adaptability at temporal high-rate monopolization market state changes

- Rate, efficiency, optimal selection corrective government strategy and time selection define the government adaptability.

- At this stage, along with the described questions, the main results of the research were: creation of a system of government adaptability degree indicators [1, 2]; the development of program for the automated adaptability indicators calculation for specific examples [3]; the Development of statistical tools for the expert survey [7].

- Questions to be addressed further: proof the proposition that weighted sums of the optimal in pure strategies matrices are the optimal in mixed strategies matrices; range of discretization probabilities s and q selection.

- Solution the indicated problem can lead to significant practical results in developing tax and tariff policy.

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ABOUT AUTHORS

KONOVALOVA, Ekaterina Dmitrievna, magister in applied mathematics and informatics, post graduate student of chair "Mathematical methods of economics and statistics".

PANYUKOV, Anatoliy Vasilevich, doctor of physical and mathematical sciences, professor, head of chair "Mathematical methods of economics and statistics".

МЕТАДАННЫЕ

Название: Государственное управление равновесием по Штакельбергу на рынке естественной монополии.

Авторы: Е. Д. Коновалова¹, А. В. Панюков²

Организация:

ФГБОУ ВПО НИУ «Южно-Уральский государственный университет» (ЮУрГУ), Россия.

Email: ¹konovalova_ekaterina@bk.ru, ²a_panyukov@mail.ru .

Язык: английский.

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Аннотация: Исследование направлено на увеличение эффективности методик государственного регулирования рынка естественной монополии. Проблема сформулирована в виде двух задач: построение модели взаимодействия монополиста и покупателей на рынке; построение модели взаимодействия государства с рынком. Рынок формализован в виде некооперативной неантагонистической игры двух лиц. На рынке устанавливается равновесие по Штакельбергу в смешанных стратегиях. Государство воздействует на установившееся равновесие неценовыми рычагами путем задания корректирующих матриц. Решается задача поиска оптимальных корректирующих государственных политик.

Ключевые слова: рынок естественной монополии; государственное регулирование; адаптивность; бескоалиционная игра; равновесие по Штакельбергу; смешанные стратегии.

Об авторах:

КОНОВАЛОВА Екатерина Дмитриевна, асп. каф. экономико-математических методов и статистики. М-р прикл. мат. и инф. (ЮУрГУ, 2012).

ПАНЮКОВ, Анатолий Васильевич, зав. каф. экономико-математических методов и статистики. Д-р физ.-мат. наук, (ВЦ РАН им. А. А. Дродницына, 2000), проф. Иссл. в обл. оптимизации, мат. моделирования в естествознании, технике, экономике, доказательных вычислениях, программировании.